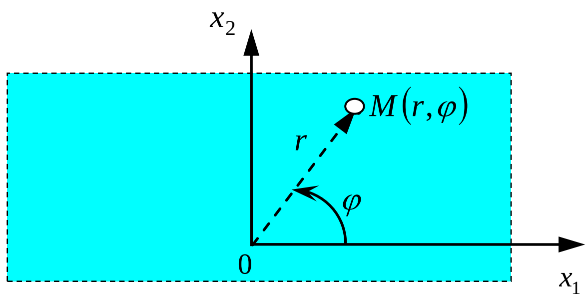
List of the Boundary-value Problem (P2Ps) for canonical polar domains, for which the Green's Functions for Poisson Equation have been derived

All these Green's functions can be found in the book:

Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

*(P2P) **1. Plane*** $(0 \le r \le \infty,\ 0 \le \varphi \le 2\pi)$



Boundary-value Problem (P2P)

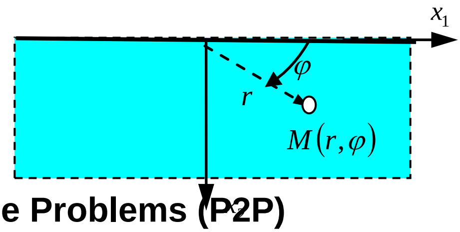
Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation in the inner points of the no limited plane.

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$

$$\nabla^2 U(r,\varphi) = -f(r,\varphi).$$

Answer to Problem (P2P) 1 for plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 2. Half-plane $(0 \le r \le \infty,\ 0 \le \varphi \le \pi)$



Boundary-value Problems (P2P)

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson's equations

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi)$$

in the inner points of the half-plane. On its boundaries are given and the following boundary conditions:

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$U(r,\rho) = \int_0^\pi \int_0^\infty f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho\, d\rho\, d\psi +$$
$$\int_0^\infty \left[\frac{\partial U(\rho,\pi)}{\partial n_1} G(\rho,r;\pi,\varphi) - U(\rho,\pi) \frac{\partial G(\rho,r;\pi,\varphi)}{\partial n_1} \right] d\rho + \qquad (2.3)$$
$$\int_0^\infty \left[\frac{\partial U(\rho,0)}{\partial n_1} G(\rho,r;0,\varphi) - U(\rho,0) \frac{\partial G(\rho,r;0,\varphi)}{\partial n_1} \right] d\rho.$$

In eq.(2.3) Green functions G and functions U satisfy Poisson Equations

Problems (P2P) 2.1- (P2P) 2.4

Problem (P2P) 2.1

$$G^{(1)} = 0;\ \ \varphi = 0;\ \ 0 \le r \le \infty; U(\rho,0) = s_1(\rho,0);$$
$$G^{(1)} = 0;\ \ \varphi = \pi;\ \ 0 \le r \le \infty; U(\rho,\pi) = s_2(\rho,\pi).$$

Answer to Problem (P2P) 2.1 for half-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 2.2

$$\partial G^{(2)} \big/ \partial n_1 = 0; \;\; \varphi = 0; \;\; 0 \le r \le \infty; \partial U(\rho,0)/\partial n_1 = g_1(\rho,0);$$
$$\partial G^{(2)} \big/ \partial n_1 = 0; \;\; \varphi = \pi; \;\; 0 \le r \le \infty; \partial U(\rho,\pi)/\partial n_1 = g_2(\rho,\pi);$$

Answer to Problem (P2P) 2.2 for half-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 2.3

$$G^{(3)} = 0, \; \varphi = 0; \;\; 0 \le r \le \infty; \quad U(\rho,0) = s_1(\rho,0);$$
$$\partial G^{(3)} \big/ \partial n_1 = 0; \;\; \varphi = \pi; \;\; 0 \le r \le \infty; \partial U(\rho,\pi)/\partial n_1 = g_2(\rho,\pi)$$

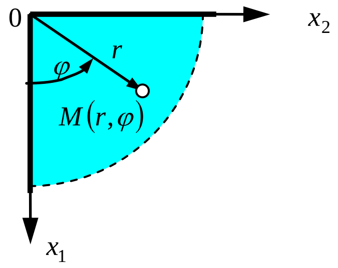
Answer to Problem (P2P) 2.3 for half-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 2.4

$$\partial G^{(4)} \big/ \partial n_1 = 0, \; \varphi = 0; \;\; 0 \le r \le \infty; \partial U(\rho,0)/\partial n_1 = g_1(\rho,0);$$
$$G^{(4)} = 0; \;\; \varphi = \pi; \;\; 0 \le r \le \infty; \quad U(\rho,\pi) = s_2(\rho,\pi)$$

Answer to Problem (P2P) 2.4 for half-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 3. Quarter-plane $(0 \le r \le \infty, \; 0 \le \varphi \le \pi/2)$



Boundary-value Problems (P2P)

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi)$$

in the inner points of the quarter of plane (quadrant). On its boundaries are given the following boundary conditions:

$$\begin{aligned}\partial U(\rho,0)/\partial n_1 = g_1(\rho,0) \;\; or \;\; U(\rho,0) = s_1(\rho,0) \;\; and \\ \partial U(\rho,\pi/2)/\partial n_2 = g_2(\rho,\alpha) \;\; or \;\; U(\rho,\alpha) = s_2(\rho,\pi/2).\end{aligned} \qquad (3.2)$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{aligned}U(r,\rho) = \int_0^{\pi/2}\int_0^{\infty} f(\rho,\psi)G(r,\rho;\varphi,\psi)\rho d\rho d\psi + \\ \int_0^{\infty}\left[\frac{\partial U(\rho,0)}{\partial n_2}G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_2}\right]d\rho + \\ \int_0^{\infty}\left[\frac{\partial U(\rho,\pi/2)}{\partial n_1}G(\rho,r;\pi/2,\varphi) - U(\rho,\pi/2)\frac{\partial G(\rho,r;\pi/2,\varphi)}{\partial n_1}\right]d\rho\end{aligned} \qquad (3.3)$$

Problems (P2P) 3.1— (P2P) 3.4

Problem (P2P) 3.1

$$\partial G^{(1)}/\partial n_2 = 0;\ \varphi = 0,\quad 0 \le r \le \infty;\quad \partial U(\rho,0)/\partial n_2 = g_1(\rho,0);$$
$$\partial G^{(1)}/\partial n_1 = 0; \varphi = \pi/2;\ 0 \le r \le \infty;\ \partial U(\rho,\pi/2)/\partial n_1 = g_2(\rho,\pi/2)$$

Answer to Problem (P2P) 3.1 for quarter-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 3.2

$$G^{(2)} = 0;\ \varphi = 0,\ 0 \le r \le \infty;\quad U(\rho,0) = s_1(\rho,0);$$
$$G^{(2)} = 0;\ \varphi = \pi/2,\ 0 \le r \le \infty;\quad U(\rho,\pi/2) = s_2(\rho,\pi/2)$$

Answer to Problem (P2P) 3.2 for quarter-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 3.3

$$\partial G^{(3)}/\partial n_2 = 0; \varphi = 0,\ 0 \le r \le \infty;\quad \partial U(\rho,0)/\partial n_2 = g_1(\rho,0);$$
$$G^{(3)} = 0,\ \varphi = \pi/2;\ 0 \le r \le \infty;\quad U(\rho,\pi/2) = s_2(\rho,\pi/2)$$

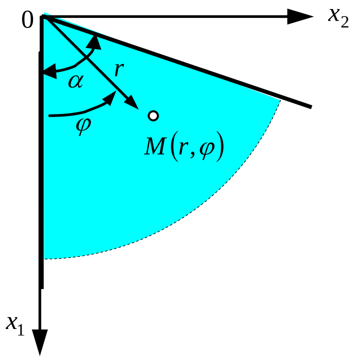
Answer to Problem (P2P) 3.3 for quarter-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 3.4

$$G^{(4)} = 0,\ \varphi = 0;\ 0 \le r \le \infty;\quad U(\rho,0) = s_1(\rho,0);$$
$$\partial G^{(4)}/\partial n_2 = 0; \varphi = \pi/2,\ 0 \le r \le \infty;\quad \partial U(\rho,\pi/2)/\partial n_2 = g_2(\rho,\pi/2)$$

Answer to Problem (P2P) 3.4 for quarter-plane (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 4. Unlimited angle $(0 \le r < \infty; 0 \le \varphi \le \alpha; \quad 0 < \alpha < \pi)$



4.1 Boundary-value Problem (P2P)s and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{4.1}$$

in the inner points of the quarter of plane (quadrant). On the boundaries are given two of the following four functions:

$$\begin{gathered} \partial U(\rho,0)/\partial n_0 = g_1(\rho,0) \;\; or \;\; U(\rho,0) = s_1(\rho,0) \quad and \\ \partial U(\rho,\alpha)/\partial n_\alpha = g_2(\rho,\alpha) \quad or \quad U(\rho,\alpha) = s_2(\rho,\alpha). \end{gathered} \tag{4.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$U(r,\rho)=\int_0^{\alpha}\int_0^{\infty} f(\rho,\psi)G(r,\rho;\varphi,\psi)\rho d\rho d\psi +$$

$$\int_0^{\infty}\left[\frac{\partial U(\rho,0)}{\partial n_0}G(\rho,r;0,\varphi)-U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_0}\right]d\rho + \tag{4.3}$$

$$\int_0^{\infty}\left[\frac{\partial U(\rho,\alpha)}{\partial n_\alpha}G(\rho,r;\alpha,\varphi)-U(\rho,\alpha)\frac{\partial G(\rho,r;\alpha,\varphi)}{\partial n_\alpha}\right]d\rho$$

In eq.(4.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi)=-\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi)=-f(r,\varphi) \tag{4.4}$$

and the following boundary conditions:

Problems (P2P) 4.1— (P2P) 4.4

Problem (P2P) 4.1

$$\partial G^{(1)}/\partial n_0=0;\ \varphi=0;\ 0\le r\le\infty;\quad \partial U(\rho,0)/\partial n_0=g_1(\rho,0);$$
$$\partial G^{(1)}/\partial n_\alpha=0;\ \varphi=\alpha;\ 0\le r\le\infty;\quad \partial U(\rho,\alpha)/\partial n_\alpha=g_2(\rho,\alpha)$$

Answer to Problem (P2P) 4.1 for unlimited-angle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 4.2

$$G^{(2)}=0;\ \varphi=0,\ 0\le r\le\infty;\quad U(\rho,0)=s_1(\rho,0);$$
$$G^{(2)}=0;\varphi=\alpha;\quad 0\le r\le\infty;\quad U(\rho,\alpha)=s_2(\rho,\alpha)$$

Answer to Problem (P2P) 4.2 for unlimited-angle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 4.3

$$\begin{array}{|l|}\hline \partial G^{(3)}/\partial n_0 = 0;\ \varphi = 0,\ 0 \le r \le \infty; \quad \partial U(\rho,0)/\partial n_0 = g_1(\rho,0) \\ G^{(3)} = 0;\ \ \varphi = \alpha,\ \ 0 \le r \le \infty; \quad U(\rho,\alpha) = s_2(\rho,\alpha). \\ \hline \end{array}$$

Answer to Problem (P2P) 4.3 for unlimited-angle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

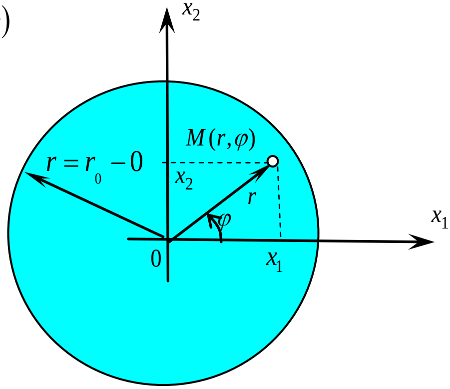
Problem (P2P) 4.4

$$\begin{array}{|c|}\hline G^{(4)} = 0;\ \ \varphi = 0,\ \ 0 \le r \le \infty; \quad U(\rho,0) = s_1(\rho,0); \\ \partial G^{(4)}/\partial n_\alpha = 0;;\ \ \varphi = \alpha,\ \ 0 \le r \le \infty; \partial U(\rho,\alpha)/\partial n_\alpha = g_2(\rho,\alpha) \\ \hline \end{array}$$

Answer to Problem (P2P) 4.4 for unlimited-angle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 5. The interior of the circle

$(0 \le r \le r_0 - 0, \quad 0 \le \varphi \le 2\pi)$



5.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{5.1}$$

in the inner points of the circle. On the it boundary are given one of the following two functions:

$$\partial U(r_0,\varphi') / \partial n_{r_0} = g_1(r_0,\varphi') \; or \; U(r_0,\varphi') = s_1(r_0,\varphi') \tag{5.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$U(r,\rho) = \int_0^{2\pi}\int_0^{r_0} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\psi + \int_0^{2\pi}\left[\frac{\partial U(r_0,\varphi')}{\partial n_{r_0}} G(r_0,r;\varphi',\varphi) - U(r_0,\varphi')\frac{\partial G(r_0,r;\varphi',\varphi)}{\partial n_{r_0}}\right] r_0 d\varphi' . \tag{5.3}$$

In eq.(5.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{5.4}$$

and the following boundary conditions:

Problems (P2P) 5.1- (P2P) 5.2

Problem (P2P) 5.1 $\boxed{G=0;\ r=r_0-0,\ 0\le\varphi\le 2\pi;\quad U(r_0,\varphi')=s_1(\rho,0)}$

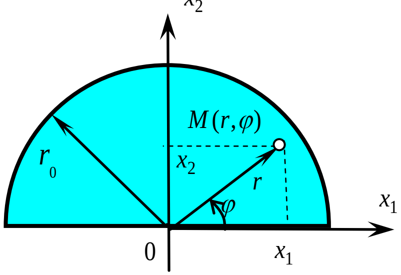
Answer to Problem (P2P) 5.1 for interior of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 5.2

$\boxed{\partial G/\partial n_{r_0}=0,\quad r=r_0-0,\ 0\le\varphi\le 2\pi;\quad \partial U(r_0,\varphi')/\partial n_{r_0}=g_1(\rho,0)}$

Answer to Problem (P2P) 5.2 for interior of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 6. Half of the circle $(r_0+0\le r<\infty,\quad 0\le\varphi\le\pi)$



6.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{6.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{gathered}
\partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0); \\
\partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_2(\rho,\pi) \quad or \quad U(\rho,\pi) = s_2(\rho,\pi) \quad and \\
\partial U(r_0,\varphi^/)/\partial n_{r_0} = g_3(r_0,\varphi^/) \; or \; U(r_0,\varphi^/) = s_3(r_0,\varphi^/).
\end{gathered} \tag{6.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}
U(r,\rho) = \int_0^\pi \int_0^{r_0} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\psi + \\
\int_0^{r_0} \left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0) \frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}} \right] d\rho + \\
\int_0^{r_0} \left[\frac{\partial U(\rho,\pi)}{\partial n_{\varphi_\pi}} G(\rho,r;\pi,\varphi) - U(\rho,\pi) \frac{\partial G(\rho,r;\pi,\varphi)}{\partial n_{\varphi_\pi}} \right] d\rho + \\
\int_0^{\pi} \left[\frac{\partial U(r_0,\varphi^/)}{\partial n_{r_0}} G(r_0,r;\varphi^/,\varphi) - U(r_0,\varphi^/) \frac{\partial G(r_0,r;\varphi^/,\varphi)}{\partial n_{r_0}} \right] r_0 d\varphi^/
\end{gathered} \tag{6.3}$$

In eq.(6.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{gathered}
\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi); \\
\nabla^2 U(r,\varphi) = -f(r,\varphi)
\end{gathered} \tag{6.4}$$

and the following boundary conditions:

Problems (P2P) 6.1- (P2P) 6.8

Problem (P2P) 6.1

$$\begin{array}{|c|}\hline G^{(1)} = 0, \quad r = r_0, \quad 0 \le \varphi \le \pi; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\ G^{(1)} = 0, \quad \varphi = 0, \quad 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0); \\ G^{(1)} = 0, \quad \varphi = \pi, \quad 0 \le r \le r_0, \quad U(\rho, \pi) = s_2(\rho, \pi). \\ \hline\end{array}$$

Answer to Problem (P2P) 6.1 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.2

$$\begin{array}{|c|}\hline \partial G^{(2)} / \partial n_{r_0} = 0, r = r_0 - 0, 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/}) / \partial n_{r_0} = g_3(r_0, \varphi^{/}); \\ \partial G^{(2)} / \partial n_{\varphi_0} = 0, \varphi = 0, 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(2)} / \partial n_{\varphi_\pi} = 0, \varphi = \pi, 0 \le r \le r_0, \quad \partial U(\rho, \pi) / \partial n_{\varphi_\pi} = g_2(\rho, \pi). \\ \hline\end{array}$$

Answer to Problem (P2P) 6.2 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.3

$$\begin{array}{|c|}\hline G^{(3)} = 0, r = r_0 - 0, 0 \le \varphi \le \pi; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\ \partial G^{(3)} / \partial n_{\varphi_0} = 0, \varphi = 0, 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(3)} / \partial n_{\varphi_\pi} = 0, \varphi = \pi, 0 \le r \le r_0; \quad \partial U(\rho, \pi) / \partial n_{\varphi_\pi} = g_2(\rho, \pi). \\ \hline\end{array}$$

Answer to Problem (P2P) 6.3 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.4

$$
\begin{array}{c}
G^{(4)} = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\
\partial G^{(4)} / \partial n_{\varphi_0} = 0, \varphi = 0,\, 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(4)} = 0,\, \varphi = \pi,\, 0 \le r \le r_0; \quad U(\rho, \pi) = s_2(\rho, \pi).
\end{array}
$$

Answer to Problem (P2P) 6.4 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.5

$$
\begin{array}{c}
G^{(5)} = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\
G^{(5)} = 0, \varphi = 0,\, 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0); \\
\partial G^{(5)} / \partial n_{r_0} ,\, \varphi = \pi,\, 0 \le r \le r_0; \quad \partial U(\rho, \pi) / \partial n_{\varphi_\pi} = g_2(\rho, \pi).
\end{array}
$$

Answer to Problem (P2P) 6.5 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.6

$$
\begin{array}{c}
\partial G^{(6)} / \partial n_{r_0} = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/}) / \partial n_{r_0} = g_3(r_0, \varphi^{/}); \\
\partial G^{(6)} / \partial n_{\varphi_0} = 0, \varphi = 0,\, 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(6)} = 0,\, \varphi = \pi,\, 0 \le r \le r_0; \quad U(\rho, \pi) = s_2(\rho, \pi).
\end{array}
$$

Answer to Problem (P2P) 6.6 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 6.7

$$\partial G^{(7)}/\partial n_{r_0} = 0, r = r_0 - 0, 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(7)} = 0, \varphi = 0, 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0, \varphi = \pi, 0 \le r \le r_0; \quad U(\rho, \pi) = s_2(\rho, \pi).$$

Answer to Problem (P2P) 6.7 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

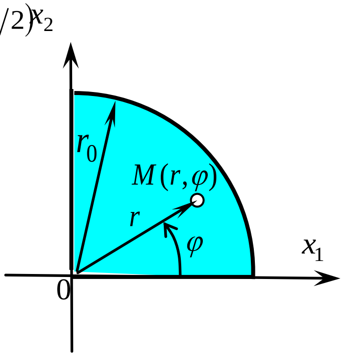
Problem (P2P) 6.8

$$\partial G^{(8)}/\partial n_{r_0} = 0, r = r_0 - 0, 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(8)} = 0, \varphi = 0, 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)}/\partial n_{\varphi_\pi} = 0, \varphi = \pi, 0 \le r \le r_0; \quad \partial U(\rho, \pi)/\partial n_{\varphi_\pi} = g_2(\rho, \pi).$$

Answer to Problem (P2P) 6.8 for half of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 7. Quarter of the circle

$(r_0 + 0 \le r < \infty, \quad 0 \le \varphi \le \pi/2)$



7.1 Boundary-value Problem (P2P)s and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{7.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{gathered}\partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0);\\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}} = g_2(\rho,\pi/2) \quad or \quad U(\rho,\pi/2) = s_2(\rho,\pi/2) \quad and\\ \partial U(r_0,\varphi^{/})/\partial n_{r_0} = g_3(r_0,\varphi^{/}) \; or \; U(r_0,\varphi^{/}) = s_3(r_0,\varphi^{/}).\end{gathered} \tag{7.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}U(r,\rho) = \int_0^{\pi/2}\int_0^{r_0} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho\, d\rho\, d\psi + \\ \int_0^{r_0}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right] d\rho + \\ \int_0^{r_0}\left[\frac{\partial U(\rho,\pi/2)}{\partial n_{\varphi_{\pi/2}}} G(\rho,r;\pi/2,\varphi) - U(\rho,\pi/2)\frac{\partial G(\rho,r;\pi/2,\varphi)}{\partial n_{\varphi_{\pi/2}}}\right] d\rho + \\ \int_0^{\pi/2}\left[\frac{\partial U(r_0,\varphi^{/})}{\partial n_{r_0}} G(r_0,r;\varphi^{/},\varphi) - U(r_0,\varphi^{/})\frac{\partial G(r_0,r;\varphi^{/},\varphi)}{\partial n_{r_0}}\right] r_0\, d\varphi^{/}\end{gathered} \tag{7.3}$$

In eq.(7.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{gathered}\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);\\ \nabla^2 U(r,\varphi) = -f(r,\varphi)\end{gathered} \tag{7.4}$$

and the following boundary conditions:

Problems (P2P) 7.1- (P2P) 7.8

Problem (P2P) 7.1

$$\boxed{\begin{array}{c} G^{(1)} = 0, r = r_0 - 0, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\ G^{(1)} = 0, \varphi = 0, 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0); \\ G^{(1)} = 0, \varphi = \pi/2, 0 \le r \le r_0; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2). \end{array}}$$

Answer to Problem (P2P) 7.1 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.2

$$\boxed{\begin{array}{c} \partial G^{(2)}\big/\partial n_{r_0} = 0, r = r_0 - 0, 0 \le \varphi \le \pi/2; \quad \partial U\left(r_0, \varphi^{/}\right)\big/\partial n_{r_0} = g_3\left(r_0, \varphi^{/}\right); \\ \partial G^{(2)}\big/\partial n_{\varphi_0} = 0, \varphi = 0, 0 \le r \le r_0; \quad \partial U(\rho, 0)\big/\partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(2)}\big/\partial n_{\varphi_{\pi/2}} = 0, \varphi = \pi/2, 0 \le r \le r_0; \quad \partial U(\rho, \pi/2)\big/\partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2). \end{array}}$$

Answer to Problem (P2P) 7.2 for quarter of the circle (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 7.3

$$\boxed{\begin{array}{c} G^{(3)} = 0, r = r_0 - 0, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\ \partial G^{(3)}\big/\partial \varphi = 0, \varphi = 0, 0 \le r \le r_0; \quad \partial U(\rho, 0)\big/\partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(3)}\big/\partial \varphi = 0, \varphi = \pi/2, 0 \le r \le r_0; \quad \partial U(\rho, \pi/2)\big/\partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2). \end{array}}$$

Answer to Problem (P2P) 7.3 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.4

$$\begin{array}{|c|}\hline G^{(4)} = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\ \partial G^{(4)} \big/ \partial n_{\varphi_0} = 0, \varphi = 0,\, 0 \le r \le r_0; \quad \partial U(\rho,0) \big/ \partial n_{\varphi_0} = g_1(\rho,0); \\ G^{(4)} = 0, \varphi = \pi/2,\, 0 \le r \le r_0; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2). \\ \hline \end{array}$$

Answer to Problem (P2P) 7.4 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.5

$$\begin{array}{|c|}\hline G^{(5)} = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\ G^{(5)} = 0, \varphi = 0,\, 0 \le r \le r_0; \quad U(\rho,0) = s_1(\rho,0); \\ \partial G^{(5)} \big/ \partial n_{\varphi_{\pi/2}},\, \varphi = \pi/2,\, 0 \le r \le r_0; \quad \partial U(\rho, \pi/2) \big/ \partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2). \\ \hline \end{array}$$

Answer to Problem (P2P) 7.5 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.6

$$\begin{array}{|c|}\hline \partial G^{(6)} \big/ \partial r = 0,\, r = r_0 - 0,\, 0 \le \varphi \le \pi/2; \quad \partial U\left(r_0, \varphi^{/}\right) \big/ \partial n_{r_0} = g_3\left(r_0, \varphi^{/}\right); \\ \partial G^{(6)} \big/ \partial \varphi = 0, \varphi = 0,\, 0 \le r \le r_0; \quad \partial U(\rho,0) \big/ \partial n_{\varphi_0} = g_1(\rho,0); \\ G^{(6)} = 0, \varphi = \pi/2,\, 0 \le r \le r_0; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2). \\ \hline \end{array}$$

Answer to Problem (P2P) 7.6 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.7

$$\partial G^{(7)} / \partial n_{r_0} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \pi/2; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(7)} = 0,\ \varphi = 0,\ 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0,\ \varphi = \pi/2,\ 0 \le r \le r_0; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2).$$

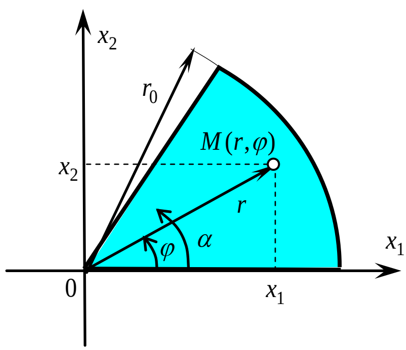
Answer to Problem (P2P) 7.7 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 7.8

$$\partial G^{(8)} / \partial n_{r_0} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \pi/2; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(8)} = 0,\ \varphi = 0,\ 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)} / \varphi_{\pi/2} = 0,\ \varphi = \pi/2,\ 0 \le r \le r_0; \quad \partial U(\rho, \pi/2) / \partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2).$$

Answer to Problem (P2P) 7.8 for quarter of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) ***8. Circular sector*** $(0 \le r \le r_0 - 0, \quad 0 \le \varphi \le \alpha)$



8.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{8.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{gathered}
\partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0); \\
\partial U(\rho,\alpha)/\partial n_{\varphi_\alpha} = g_2(\rho,\alpha) \quad or \quad U(\rho,\alpha) = s_2(\rho,\alpha) \quad and \\
\partial U(r_0,\varphi^{/})/\partial n_{r_0} = g_3(r_0,\varphi^{/}) \ or \ U(r_0,\varphi^{/}) = s_3(r_0,\varphi^{/}).
\end{gathered} \tag{8.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}
U(r,\varphi) = \int_0^{\alpha}\int_0^{r_0} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\psi + \\
\int_0^{r_0}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right] d\rho + \\
\int_0^{r_0}\left[\frac{\partial U(\rho,\alpha)}{\partial n_{\varphi_\alpha}} G(\rho,r;\alpha,\varphi) - U(\rho,\alpha)\frac{\partial G(\rho,r;\alpha,\varphi)}{\partial n_{\varphi_\alpha}}\right] d\rho + \\
\int_0^{\alpha}\left[\frac{\partial U(r_0,\varphi^{/})}{\partial n_{r_0}} G(r_0,r;\varphi^{/},\varphi) - U(r_0,\varphi^{/})\frac{\partial G(r_0,r;\varphi^{/},\varphi)}{\partial n_{r_0}}\right] r_0 d\varphi^{/}
\end{gathered} \tag{8.3}$$

In eq.(8.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{gathered}
\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi); \\
\nabla^2 U(r,\varphi) = -f(r,\varphi)
\end{gathered} \tag{8.4}$$

and the following boundary conditions:

Problems (P2P) 8.1- (P2P) 8.8

Problem (P2P) 8.1

$$\begin{array}{c} G^{(1)} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \alpha; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\ G^{(1)} = 0, \varphi = 0,\ 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0); \\ G^{(1)} = 0,\ \varphi = \alpha,\ 0 \le r \le r_0; \quad U(\rho, \alpha) = s_2(\rho, \alpha). \end{array}$$

Answer to Problem (P2P) 8.1 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.2

$$\begin{array}{c} \partial G^{(2)} / \partial n_{r_0} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \alpha; \quad \partial U(r_0, \varphi^{/}) / \partial n_{r_0} = g_3(r_0, \varphi^{/}); \\ \partial G^{(2)} / \partial n_{\varphi_0} = 0, \varphi = 0,\ 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(2)} / \partial n_{\varphi_\alpha} = 0,\ \varphi = \alpha,\ 0 \le r \le r_0; \quad \partial U(\rho, \alpha) / \partial n_{\varphi_\alpha} = g_2(\rho, \alpha). \end{array}$$

Answer to Problem (P2P) 8.2 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.3

$$\begin{array}{c} G^{(3)} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \alpha; \quad U(r_0, \varphi^{/}) = s_3(r_0, \varphi^{/}); \\ \partial G^{(3)} / \partial \varphi = 0, \varphi = 0,\ 0 \le r \le r_0; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\ \partial G^{(3)} / \partial \varphi = 0,\ \varphi = \alpha,\ 0 \le r \le r_0; \quad \partial U(\rho, \alpha) / \partial n_{\varphi_\alpha} = g_2(\rho, \alpha). \end{array}$$

Answer to Problem (P2P) 8.3 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.4

$$\begin{gathered} G^{(4)}=0,\, r=r_0-0,\, 0\le\varphi\le\alpha; \quad U\left(r_0,\varphi'\right)=s_3\left(r_0,\varphi'\right); \\ \partial G^{(4)}\big/\partial n_{\varphi_0}=0,\varphi=0,\, 0\le r\le r_0; \quad \partial U(\rho,0)\big/\partial n_{\varphi_0}=g_1(\rho,0); \\ G^{(4)}=0,\, \varphi=\alpha,\, 0\le r\le r_0; \quad U(\rho,\alpha)=s_2(\rho,\alpha). \end{gathered}$$

Answer to Problem (P2P) 8.4 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.5

$$\begin{gathered} G^{(5)}=0,\, r=r_0-0,\, 0\le\varphi\le\alpha; \quad U\left(r_0,\varphi'\right)=s_3\left(r_0,\varphi'\right); \\ G^{(5)}=0,\varphi=0,\, 0\le r\le r_0; \quad U(\rho,0)=s_1(\rho,0); \\ \partial G^{(5)}\big/\partial n_{\varphi_\alpha}\,,\, \varphi=\alpha,\, 0\le r\le r_0; \quad \partial U(\rho,\alpha)\big/\partial n_{\varphi_\alpha}=g_2(\rho,\alpha). \end{gathered}$$

Answer to Problem (P2P) 8.5 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.6

$$\begin{gathered} \partial G^{(6)}\big/\partial n_{r_0}=0,\, r=r_0-0,\, 0\le\varphi\le\alpha; \quad \partial U\left(r_0,\varphi'\right)\big/\partial n_{r_0}=g_3\left(r_0,\varphi'\right); \\ \partial G^{(6)}\big/\partial n_{\varphi_0}=0,\varphi=0,\, 0\le r\le r_0; \quad \partial U(\rho,0)\big/\partial n_{\varphi_0}=g_1(\rho,0) \\ G^{(6)}=0,\, \varphi=\alpha,\, 0\le r\le r_0; \quad U\left(r_0,\varphi'\right)=s_2\left(r_0,\varphi'\right). \end{gathered}$$

Answer to Problem (P2P) 8.6 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 8.7

$$\partial G^{(7)} / \partial n_{r_0} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \alpha; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(7)} = 0,\ \varphi = 0,\ 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0,\ \varphi = \alpha,\ 0 \le r \le r_0; \quad U(\rho, \alpha) = s_2(\rho, \alpha).$$

Answer to Problem (P2P) 8.7 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

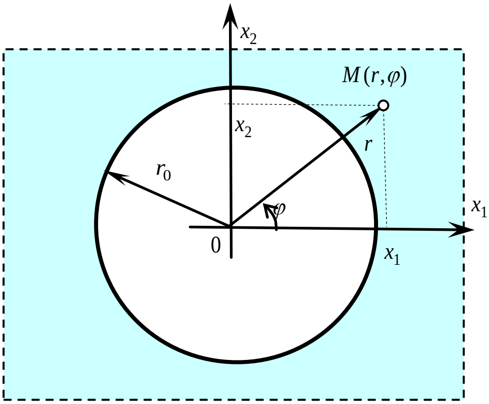
Problem (P2P) 8.8

$$\partial G^{(8)} / \partial n_{r_0} = 0,\ r = r_0 - 0,\ 0 \le \varphi \le \alpha; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(8)} = 0,\ \varphi = 0,\ 0 \le r \le r_0; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)} / \partial \varphi = 0,\ \varphi = \alpha,\ 0 \le r \le r_0; \quad \partial U(\rho, \alpha) / \partial n_{\varphi_\alpha} = g_2(\rho, \alpha).$$

Answer to Problem (P2P) 8.8 for circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 9. The exterior of the circle

$\left(r_0 + 0 \le r < \infty, \quad 0 \le \varphi \le 2\pi\right)$



9.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{9.1}$$

in the external points of the circle. On the it boundary is given one of the following two functions:

$$\partial U\left(r_0,\varphi^{/}\right)/\partial n_{r_0} = g_1\left(r_0,\varphi^{/}\right) \; or \; U\left(r_0,\varphi^{/}\right) = s_1\left(r_0,\varphi^{/}\right) \tag{9.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$U(r,\rho) = \int_0^{2\pi}\int_{r_0}^{\infty} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho\, d\rho\, d\psi + \int_0^{2\pi}\left[\frac{\partial U\left(r_0,\varphi^{/}\right)}{\partial n_{r_0}} G\left(r_0,r;\varphi^{/},\varphi\right) - U\left(r_0,\varphi^{/}\right)\frac{\partial G\left(r_0,r;\varphi^{/},\varphi\right)}{\partial n_{r_0}}\right] r_0\, d\varphi^{/}. \tag{9.3}$$

In eq.(9.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{9.4}$$

and the following boundary conditions:

Problems (P2P) 9.1- (P2P) 9.2

Problem (P2P) 9.1 $\boxed{G^{(1)} = 0;\ r = r_0 + 0,\ 0 \le \varphi \le 2\pi; \quad U(r_0, \varphi') = s_1(r_0, \varphi')}$

Answer to Problem (P2P) 9.1 for exterior of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

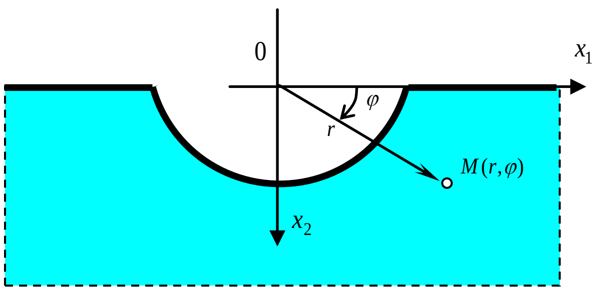
Problem (P2P) 9.2

$$\boxed{\partial G^{(2)}/\partial r = 0, \quad r = r_0 + 0,\ 0 \le \varphi \le 2\pi; \quad \partial U(r_0, \varphi')/\partial n_{r_0} = g_1(r_0, \varphi')}$$

Answer to Problem (P2P) 9.2 for exterior of the circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) ***10. Half-plane excluding half-circle***

$(r_0 \le r < \infty;\ 0 \le \varphi \le \pi)$



10.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{10.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{aligned}
&\partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0);\\
&\partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_2(\rho,\pi) \quad or \quad U(\rho,\pi) = s_2(\rho,\pi) \quad and\\
&\partial U(r_0,\varphi^/)/\partial n_{r_0} = g_3(r_0,\varphi^/) \;or\; U(r_0,\varphi^/) = s_3(r_0,\varphi^/).
\end{aligned} \tag{10.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{aligned}
U(r,\rho) = &\int_0^\pi \int_0^\infty f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\psi + \\
&\int_0^\infty \left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}} \right] d\rho + \\
&\int_{-\infty}^0 \left[\frac{\partial U(\rho,\pi)}{\partial n_{\varphi_\pi}} G(\rho,r;\pi,\varphi) - U(\rho,\pi)\frac{\partial G(\rho,r;\pi,\varphi)}{\partial n_{\varphi_\pi}} \right] d\rho + \\
&\int_0^\pi \left[\frac{\partial U(r_0,\varphi^/)}{\partial n_{r_0}} G(r_0,r;\varphi^/,\varphi) - U(r_0,\varphi^/)\frac{\partial G(r_0,r;\varphi^/,\varphi)}{\partial n_{r_0}} \right] r_0 d\varphi^/
\end{aligned} \tag{10.3}$$

In eq.(10.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{aligned}
\nabla^2 G(r,\rho;\varphi,\psi) &= -\delta(r-\rho)\delta(\varphi-\psi);\\
\nabla^2 U(r,\varphi) &= -f(r,\varphi)
\end{aligned} \tag{10.4}$$

and the following boundary conditions:

Problems (P2P) 10.1- (P2P) 10.8

Problem (P2P) 10.1

$$G^{(1)} = 0, r = r_0 + 0, 0 \le \varphi \le \pi; \quad U(r_0, \varphi') = s_3(r_0, \varphi');$$
$$G^{(1)} = 0, \varphi = 0, r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(1)} = 0, \varphi = \pi, r_0 \le r < \infty; \quad U(\rho, \pi) = s_2(\rho, \pi).$$

Answer to Problem (P2P) 10.1 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.2

$$\partial G^{(2)}/\partial n_{r_0} = 0, r = r_0 + 0, 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi')/\partial n_{r_0} = g_3(r_0, \varphi');$$
$$\partial G^{(2)}/\partial \varphi = 0, \varphi = 0, r_0 \le r < \infty; \quad \partial U(\rho, 0)/\partial n_{\varphi_0} = g_1(\rho, 0);$$
$$\partial G^{(2)}/\partial n_{\varphi_\pi} = 0, \varphi = \pi, r_0 \le r < \infty; \quad \partial U(\rho, \pi)/\partial n_{\varphi_\pi} = g_2(\rho, \pi).$$

Answer to Problem (P2P) 10.2 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.3

$$G^{(3)} = 0, r = r_0 + 0, 0 \le \varphi \le \pi; \quad U(r_0, \varphi') = s_3(r_0, \varphi');$$
$$\partial G^{(3)}/\partial \varphi = 0, \varphi = 0, r_0 \le r < \infty; \quad \partial U(\rho, 0)/\partial n_{\varphi_0} = g_1(\rho, 0);$$
$$\partial G^{(3)}/\partial \varphi = 0, \varphi = \pi, r_0 \le r < \infty; \quad \partial U(\rho, \pi)/\partial n_{\varphi_\pi} = g_2(\rho, \pi).$$

Answer to Problem (P2P) 10.3 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.4

$$\begin{array}{c} G^{(4)}=0,\, r=r_0+0,\, 0\le\varphi\le\pi; \quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right); \\ \partial G^{(4)}/\partial n_{\varphi_0}=0,\varphi=0,\, r_0\le r<\infty; \quad \partial U(\rho,0)/\partial n_{\varphi_0}=g_1(\rho,0); \\ G^{(4)}=0,\,\varphi=\pi,\, r_0\le r<\infty; \quad U(\rho,\pi)=s_2(\rho,\pi). \end{array}$$

Answer to Problem (P2P) 10.4 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.5

$$\begin{array}{c} G^{(5)}=0,\, r=r_0+0,\, 0\le\varphi\le\pi; \quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right); \\ G^{(5)}=0,\varphi=0,\, r_0\le r<\infty; \quad U(\rho,0)=s_1(\rho,0); \\ \partial G^{(5)}/\partial n_{\varphi_\pi}=0,\,\varphi=\pi,\, r_0\le r<\infty; \quad \partial U(\rho,\pi)/\partial n_{\varphi_\pi}=g_2(\rho,\pi). \end{array}$$

Answer to Problem (P2P) 10.5 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.6

$$\begin{array}{c} \partial G^{(6)}/\partial n_{r_0}=0,\, r=r_0+0,\, 0\le\varphi\le\pi; \quad \partial U\left(r_0,\varphi^{/}\right)/\partial n_{r_0}=g_3\left(r_0,\varphi^{/}\right); \\ \partial G^{(6)}/\partial n_{\varphi_0}=0,\varphi=0,\, r_0\le r<\infty; \quad \partial U(\rho,0)/\partial n_{\varphi_0}=g_1(\rho,0); \\ G^{(6)}=0,\,\varphi=\pi,\, r_0\le r<\infty; \quad U(\rho,\pi)=s_2(\rho,\pi) \end{array}$$

Answer to Problem (P2P) 10.6 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 10.7

$$\partial G^{(7)}/\partial n_{r_0} = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(7)} = 0,\ \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0,\ \varphi = \pi,\ r_0 \le r < \infty; \quad U(\rho, \pi) = s_2(\rho, \pi).$$

Answer to Problem (P2P) 10.7 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

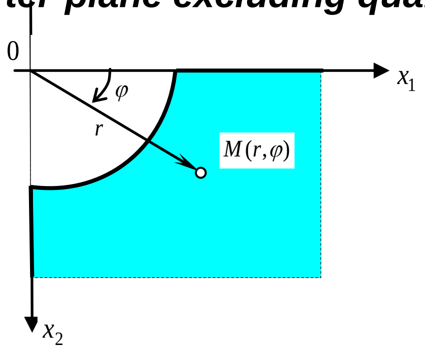
Problem (P2P) 10.8

$$\partial G^{(8)}/\partial n_{r_0} = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \pi; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(8)} = 0,\ \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)}/\partial n_{\varphi_\pi} = 0,\ \varphi = \pi,\ r_0 \le r < \infty; \quad \partial U(\rho, \pi)/\partial n_{\varphi_\pi} = g_2(\rho, \pi).$$

Answer to Problem (P2P) 10.8 for half-plane excluding half-circle (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) ***11. Quarter-plane excluding quarter-circle***

$(r_0 \le r < \infty;\ 0 \le \varphi \le \pi/2)$



11.1. Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{11.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{gathered}\partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0);\\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}} = g_2(\rho,\pi/2) \quad or \quad U(\rho,\pi/2) = s_2(\rho,\pi/2) \quad and\\ \partial U(r_0,\varphi^/)/\partial n_{r_0} = g_3(r_0,\varphi^/) \; or \; U(r_0,\varphi^/) = s_3(r_0,\varphi^/).\end{gathered} \tag{11.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}U(r,\rho) = \int_0^{\pi/2}\int_0^{\infty} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\psi +\\ \int_0^{\infty}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right] d\rho +\\ \int_0^{\infty}\left[\frac{\partial U(\rho,\pi/2)}{\partial n_{\varphi_{\pi/2}}} G(\rho,r;\pi/2,\varphi) - U(\rho,\pi/2)\frac{\partial G(\rho,r;\pi/2,\varphi)}{\partial n_{\varphi_{\pi/2}}}\right] d\rho +\\ \int_0^{\pi/2}\left[\frac{\partial U(r_0,\varphi^/)}{\partial n_{r_0}} G(r_0,r;\varphi^/,\varphi) - U(r_0,\varphi^/)\frac{\partial G(r_0,r;\varphi^/,\varphi)}{\partial n_{r_0}}\right] r_0 d\varphi^/\end{gathered} \tag{11.3}$$

In eq.(11.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{gathered}\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);\\ \nabla^2 U(r,\varphi) = -f(r,\varphi)\end{gathered} \tag{11.4}$$

and the following boundary conditions:

Problems (P2P) 11.1- (P2P) 11.8

Problem (P2P) 11.1

$$\begin{array}{|c|}\hline G^{(1)}=0,\, r=r_0+0,\, 0\le\varphi\le\pi/2;\quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right);\\ G^{(1)}=0,\varphi=0,\, r_0\le r<\infty;\quad U(\rho,0)=s_1(\rho,0);\\ G^{(1)}=0,\,\varphi=\pi/2,\, r_0\le r<\infty;\quad U(\rho,\pi/2)=s_2(\rho,\pi/2).\\ \hline\end{array}$$

Answer to Problem (P2P) 11.1 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.2

$$\begin{array}{|c|}\hline \partial G^{(2)}\big/\partial n_{r_0}=0,\, r=r_0+0,\, 0\le\varphi\le\pi/2;\quad \partial U\left(r_0,\varphi^{/}\right)\big/\partial n_{r_0}=g_3\left(r_0,\varphi^{/}\right);\\ \partial G^{(2)}\big/\partial n_{\varphi_0}=0,\varphi=0,\, r_0\le r<\infty;\quad \partial U(\rho,0)\big/\partial n_{\varphi_0}=g_1(\rho,0);\\ \partial G^{(2)}\big/\partial n_{\varphi_{\pi/2}}=0,\,\varphi=\pi/2,\, r_0\le r<\infty;\quad \partial U(\rho,\pi/2)\big/\partial n_{\varphi_{\pi/2}}=g_2(\rho,\pi/2)\\ \hline\end{array}$$

Answer to Problem (P2P) 11.2 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.3

$$\begin{array}{|c|}\hline G^{(3)}=0,\, r=r_0+0,\, 0\le\varphi\le\pi/2;\quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right);\\ \partial G^{(3)}\big/\partial\varphi=0,\varphi=0,\, r_0\le r<\infty;\quad \partial U(\rho,0)\big/\partial n_{\varphi_0}=g_1(\rho,0);\\ \partial G^{(3)}\big/\partial\varphi=0,\,\varphi=\pi/2,\, r_0\le r<\infty;\quad \partial U(\rho,\pi/2)\big/\partial n_{\varphi_{\pi/2}}=g_2(\rho,\pi/2).\\ \hline\end{array}$$

Answer to Problem (P2P) 11.3 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.4

$$
\begin{array}{|c|}
\hline
G^{(4)} = 0,\, r = r_0 + 0,\, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\
\partial G^{(4)}/\partial \varphi = 0,\, \varphi = 0,\, r_0 \le r < \infty; \quad \partial U(\rho, 0)/\partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(4)} = 0,\, \varphi = \pi/2,\, r_0 \le r < \infty; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2). \\
\hline
\end{array}
$$

Answer to Problem (P2P) 11.4 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.5

$$
\begin{array}{|c|}
\hline
G^{(5)} = 0,\, r = r_0 + 0,\, 0 \le \varphi \le \pi/2; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\
G^{(5)} = 0,\, \varphi = 0,\, r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0); \\
\partial G^{(5)}/\partial \varphi,\, \varphi = \pi/2,\, r_0 \le r < \infty; \quad \partial U(\rho, \pi/2)/\partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2). \\
\hline
\end{array}
$$

Answer to Problem (P2P) 11.5 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.6

$$
\begin{array}{|c|}
\hline
\partial G^{(6)}/\partial n_{r_0} = 0,\, r = r_0 + 0,\, 0 \le \varphi \le \pi/2; \quad \partial U\left(r_0, \varphi^{/}\right)/\partial n_{r_0} = g_3\left(r_0, \varphi^{/}\right); \\
\partial G^{(6)}/\partial n_{\varphi_0} = 0,\, \varphi = 0,\, r_0 \le r < \infty; \quad \partial U(\rho, 0)/\partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(6)} = 0,\, \varphi = \pi/2,\, r_0 \le r < \infty; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2). \\
\hline
\end{array}
$$

Answer to Problem (P2P) 11.6 for quarter-plane excluding quarter-circle

(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 11.7

$$\partial G^{(7)}/\partial n_{r_0} = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \pi/2; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(7)} = 0, \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0, \varphi = \pi/2,\ r_0 \le r < \infty; \quad U(\rho, \pi/2) = s_2(\rho, \pi/2).$$

Answer to Problem (P2P) 11.7 for quarter-plane excluding quarter-circle
(Green's function for Poisson's equation in polar coordinates)

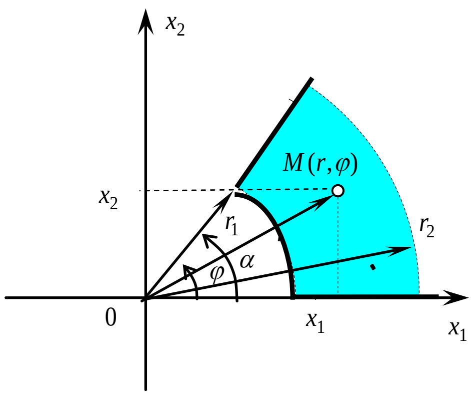
Problem (P2P) 11.8

$$\partial G^{(8)}/\partial r = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \pi/2; \quad \partial U(r_0, \varphi') / \partial n_{r_0} = g_3(r_0, \varphi');$$
$$G^{(8)} = 0, \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)}/\partial \varphi = 0, \varphi = \pi/2,\ r_0 \le r < \infty; \quad \partial U(\rho, \pi/2) / \partial n_{\varphi_{\pi/2}} = g_2(\rho, \pi/2).$$

Answer to Problem (P2P) 11.8 for quarter-plane excluding quarter-circle
(Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

(P2P) 12. Unlimited circular sector

$(r_0 + 0 \le r < \infty, \quad 0 \le \varphi \le \alpha)$



12.1. Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{12.1}$$

in the inner points of the circle. On the it boundary are given three of the following six functions:

$$\begin{gathered} \partial U(\rho,0)/\partial n_{\varphi_0} = g_1(\rho,0) \quad or \quad U(\rho,0) = s_1(\rho,0); \\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha} = g_2(\rho,\alpha) \quad or \quad U(\rho,\alpha) = s_2(\rho,\alpha) \quad and \\ \partial U(r_0,\varphi^/)/\partial n_{r_0} = g_3(r_0,\varphi^/) \; or \; U(r_0,\varphi^/) = s_3(r_0,\varphi^/). \end{gathered} \tag{12.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered} U(r,\rho) = \int_0^\alpha \int_{r_0}^\infty f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho\, d\rho\, d\psi + \\ \int_{r_0}^\infty \left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0) \frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}} \right] d\rho + \\ \int_{r_0}^\infty \left[\frac{\partial U(\rho,\alpha)}{\partial n_{\varphi_\alpha}} G(\rho,r;\alpha,\varphi) - U(\rho,\pi/2) \frac{\partial G(\rho,r;\alpha,\varphi)}{\partial n_{\varphi_\alpha}} \right] d\rho + \\ \int_0^\alpha \left[\frac{\partial U(r_0,\varphi^/)}{\partial n_{r_0}} G(r_0,r;\varphi^/,\varphi) - U(r_0,\varphi^/) \frac{\partial G(r_0,r;\varphi^/,\varphi)}{\partial n_{r_0}} \right] r_0\, d\varphi^/ \end{gathered} \tag{12.3}$$

In eq.(12.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{gathered} \nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi); \\ \nabla^2 U(r,\varphi) = -f(r,\varphi) \end{gathered} \tag{12.4}$$

and the following boundary conditions:

Problems (P2P) 12.1- (P2P) 12.8

Problem (P2P) 12.1

$$\begin{array}{|c|}\hline G^{(1)}=0,\, r=r_0+0,\, 0\le\varphi\le\alpha;\quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right); \\ G^{(1)}=0,\varphi=0,\, r_0\le r<\infty;\quad U(\rho,0)=s_1(\rho,0); \\ G^{(1)}=0,\, \varphi=\alpha,\, r_0\le r<\infty;\quad U(\rho,\alpha)=s_2(\rho,\alpha). \\ \hline\end{array}$$

Answer to Problem (P2P) 12.1 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.2

$$\begin{array}{|c|}\hline \partial G^{(2)}\big/\partial n_{r_0}=0,\, r=r_0+0,\, 0\le\varphi\le\alpha;\quad \partial U\left(r_0,\varphi^{/}\right)\big/\partial n_{r_0}=g_3\left(r_0,\varphi^{/}\right); \\ \partial G^{(2)}\big/\partial n_{\varphi_0}=0,\varphi=0,\, r_0\le r<\infty;\quad \partial U(\rho,0)\big/\partial n_{\varphi_0}=g_1(\rho,0); \\ \partial G^{(2)}\big/\partial n_{\varphi_\alpha}=0,\, \varphi=\alpha,\, r_0\le r<\infty;\quad \partial U(\rho,\alpha)\big/\partial n_{\varphi_\alpha}=g_2(\rho,\alpha). \\ \hline\end{array}$$

Answer to Problem (P2P) 12.2 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.3

$$\begin{array}{|c|}\hline G^{(3)}=0,\, r=r_0+0,\, 0\le\varphi\le\alpha;\quad U\left(r_0,\varphi^{/}\right)=s_3\left(r_0,\varphi^{/}\right); \\ \partial G^{(3)}\big/\partial\varphi=0,\varphi=0,\, r_0\le r<\infty;\quad \partial U(\rho,0)\big/\partial n_{\varphi 0}=g_1(\rho,0); \\ \partial G^{(3)}\big/\partial\varphi=0,\, \varphi=\alpha,\, r_0\le r<\infty;\quad \partial U(\rho,\alpha)\big/\partial n_{\varphi_\alpha}=g_2(\rho,\alpha). \\ \hline\end{array}$$

Answer to Problem (P2P) 12.3 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.4

$$
\begin{array}{c}
G^{(4)} = 0, r = r_0 + 0, 0 \le \varphi \le \alpha; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\
\partial G^{(4)} / \partial n_{\varphi_0} = 0, \varphi = 0, r_0 \le r < \infty; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(4)} = 0, \varphi = \alpha, r_0 \le r < \infty; \quad U(\rho, \alpha) = s_2(\rho, \alpha).
\end{array}
$$

Answer to Problem (P2P) 12.4 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.5

$$
\begin{array}{c}
G^{(5)} = 0, r = r_0 + 0, 0 \le \varphi \le \alpha; \quad U\left(r_0, \varphi^{/}\right) = s_3\left(r_0, \varphi^{/}\right); \\
G^{(5)} = 0, \varphi = 0, r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0); \\
\partial G^{(5)} / \partial n_{\varphi_\alpha}, \varphi = \alpha, r_0 \le r < \infty; \quad \partial U(\rho, \alpha) / \partial n_{\varphi_\alpha} = g_2(\rho, \alpha).
\end{array}
$$

Answer to Problem (P2P) 12.5 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.6

$$
\begin{array}{c}
\partial G^{(6)} / \partial n_{r_0} = 0, r = r_0 + 0, 0 \le \varphi \le \alpha; \quad \partial U\left(r_0, \varphi^{/}\right) / \partial n_{r_0} = g_3\left(r_0, \varphi^{/}\right); \\
\partial G^{(6)} / \partial n_{\varphi_0} = 0, \varphi = 0, r_0 \le r < \infty; \quad \partial U(\rho, 0) / \partial n_{\varphi_0} = g_1(\rho, 0); \\
G^{(6)} = 0, \varphi = \alpha, r_0 \le r < \infty; \quad U(\rho, \alpha) = s_2(\rho, \alpha).
\end{array}
$$

Answer to Problem (P2P) 12.6 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates) can be found in the book :
Victor Seremet & Guy Bonnet, Encyclopedia of Domain Green's Functions (Thermo-magneto-electrostatics of solids in rectangular and polar coordinates), State Agrarian University of Moldova: Publisher Center of UASM, Chisinau, Moldova, 2008, 220 pag., (in English).

Problem (P2P) 12.7

$$\partial G^{(7)}/\partial n_{r_0} = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \alpha; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(7)} = 0,\ \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$G^{(7)} = 0,\ \varphi = \alpha,\ r_0 \le r < \infty; \quad U(r_0, \varphi^{/}) = s_2(r_0, \varphi^{/}).$$

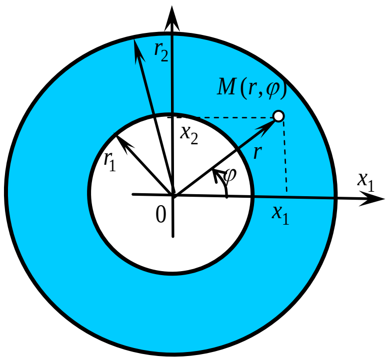
Answer to Problem (P2P) 12.7 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 12.8

$$\partial G^{(8)}/\partial n_{r_0} = 0,\ r = r_0 + 0,\ 0 \le \varphi \le \alpha; \quad \partial U(r_0, \varphi^{/})/\partial n_{r_0} = g_3(r_0, \varphi^{/});$$
$$G^{(8)} = 0,\ \varphi = 0,\ r_0 \le r < \infty; \quad U(\rho, 0) = s_1(\rho, 0);$$
$$\partial G^{(8)}/\partial n_{\varphi_\alpha} = 0,\ \varphi = \alpha,\ r_0 \le r < \infty; \quad \partial U(\rho, \alpha)/\partial n_{\varphi_\alpha} = g_2(\rho, \alpha).$$

Answer to Problem (P2P) 12.8 for unlimited circular sector (Green's function for Poisson's equation in polar coordinates)

(P2P) 13. The circular layer $(r_1 \le r \le r_2, \quad 0 \le \varphi \le 2\pi)$



13.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{13.1}$$

in the inner points of the circle. On the it boundary are given two of the following four functions:

$$\begin{aligned} &\partial U(r_{1`},\varphi^/)/\partial n_1 = g_1(r_{1`},\varphi^/) \;\; or \;\; U(r_{1`},\varphi^/) = s_1(r_{1`},\varphi^/) \; and \\ &\partial U(r_{2`},\varphi^/)/\partial n_2 = g_2(r_{2`},\varphi^/) \;\; or \;\; U(r_{2`},\varphi^/) = s_2(r_{2`},\varphi^/). \end{aligned} \tag{13.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{aligned} U(r,\varphi) = &\int_{r_1}^{r_2}\int_0^{2\pi} f(\rho,\psi)G(r,\rho;\varphi,\psi)\rho d\rho d\varphi + \\ &\int_0^{2\pi}\left[\frac{\partial U(r_1,\varphi^/)}{\partial n_1}G(r_1,r;\varphi^/,\varphi) - U(r_1,\varphi^/)\frac{\partial G(r_1,r;\varphi^/,\varphi)}{\partial n_1}\right] r_1 d\varphi^/ + \\ &\int_0^{2\pi}\left[\frac{\partial U(r_2,\varphi^/)}{\partial n_2}G(r_2,r;\varphi^/,\varphi) - U(r_2,\varphi^/)\frac{\partial G(r_2,r;\varphi^/,\varphi)}{\partial n_2}\right] r_2 d\varphi^/ \end{aligned} \tag{13.3}$$

In eq.(13.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\begin{aligned} \nabla^2 G(r,\rho;\varphi,\psi) &= -\delta(r-\rho)\delta(\varphi-\psi); \\ \nabla^2 U(r,\varphi) &= -f(r,\varphi) \end{aligned} \tag{13.4}$$

and the following boundary conditions:

Problems (P2P) 13.1- (P2P) 13.4

Problem (P2P) 13.1 $\boxed{\begin{aligned} &G^{(1)} = 0;\ r = r_1,\ 0 \le \varphi \le \infty; \quad U(r_{1`},\varphi^/) = s_1(r_{1`},\varphi^/); \\ &G^{(1)} = 0;\ \ r = r_2,\ 0 \le \varphi \le 2\pi; \quad U(r_{2`},\varphi^/) = s_2(r_{2`},\varphi^/). \end{aligned}}$

Answer to Problem (P2P) 13.1 for circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 13.2

$$\begin{array}{|l|}\hline \partial G^{(2)}/\partial n_1 = 0;\ r = r_1,\ 0 \le \varphi \le 2\pi; \quad \partial U\left(r_{1`}, \varphi'\right)/\partial n_1 = g_1\left(r_{1`}, \varphi'\right); \\ \partial G^{(2)}/\partial n_2 = 0;\ r = r_2,\ 0 \le \varphi \le \infty; \quad \partial U\left(r_{2`}, \varphi'\right)/\partial n_2 = g_2\left(r_{2`}, \varphi'\right). \\ \hline \end{array}$$

Answer to Problem (P2P) 13.2 for circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 13.3

$$\begin{array}{|c|}\hline G^{(3)} = 0;\ r = r_1,\ 0 \le \varphi \le \infty; \quad U\left(r_{1`}, \varphi'\right) = s_1\left(r_{1`}, \varphi'\right); \\ \partial G^{(3)}/\partial n_2 = 0,\ r = r_2,\ 0 \le \varphi \le 2\pi; \quad \partial U\left(r_{2`}, \varphi'\right)/\partial n_2 = g_2\left(r_{2`}, \varphi'\right). \\ \hline \end{array}$$

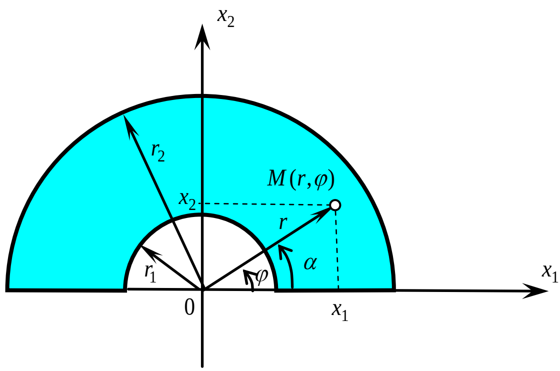
Answer to Problem (P2P) 13.3 for circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 13.4

$$\begin{array}{|c|}\hline \partial G^{(4)}/\partial n_1 = 0;\ r = r_1,\ 0 \le \varphi \le \infty; \quad \partial U\left(r_{1`}, \varphi'\right)/\partial n_1 = g_1\left(r_{1`}, \varphi'\right); \\ G^{(4)} = 0,\ r = r_2,\ 0 \le \varphi \le 2\pi; \quad U\left(r_{2`}, \varphi'\right) = s_2\left(r_{2`}, \varphi'\right). \\ \hline \end{array}$$

Answer to Problem (P2P) 13.4 for circular layer (Green's function for Poisson's equation in polar coordinates)

(P2P) ***14. Half of the circular layer*** $(r_1 \le r \le r_2;\ 0 \le \varphi \le \pi)$



14.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{14.1}$$

in the inner points of the circle. On the it boundary are given two of the following four functions:

$$\begin{gathered}
\partial U(r_{1`},\varphi') / \partial n_1 = g_1(r_{1`},\varphi') \; or \; U(r_{1`},\varphi') = s_1(r_{1`},\varphi') \, and \\
\partial U(r_{2`},\varphi') / \partial n_2 = g_2(r_2,\varphi) \quad or \quad U(r_{2`},\varphi') = s_2(r_{2`},\varphi'). \\
\partial U(\rho,0) / \partial n_{\varphi_0} = g_3(\rho,0); \quad U(\rho,0) = s_3(\rho,0); \\
\partial U(\rho,\pi) / \partial n_{\varphi_\pi} = g_4(\rho,\pi); \quad U(\rho,\pi) = s_4(\rho,\pi).
\end{gathered} \tag{14.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}
U(r,\varphi) = \int_{r_1}^{r_2}\int_0^{\pi} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\varphi + \\
\int_0^{\pi}\left[\frac{\partial U(r_1,\varphi')}{\partial n_1} G(r_1,r;\varphi',\varphi) - U(r_1,\varphi')\frac{\partial G(r_1,r;\varphi',\varphi)}{\partial n_1}\right] r_1 d\varphi' + \\
\int_0^{\pi}\left[\frac{\partial U(r_2,\varphi')}{\partial n_2} G(r_2,r;\varphi',\varphi) - U(r_2,\varphi')\frac{\partial G(r_2,r;\varphi',\varphi)}{\partial n_2}\right] r_2 d\varphi' + \\
\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right] d\rho + \\
\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,\pi)}{\partial n_{\varphi_\pi}} G(\rho,r;\pi,\varphi) - U(\rho,\pi)\frac{\partial G(\rho,r;\pi,\varphi)}{\partial n_{\varphi_\pi}}\right] d\rho.
\end{gathered} \tag{14.3}$$

In eq.(14.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \qquad (14.4)$$

and the following boundary conditions:

Problems (P2P) 14.1- (P2P) 14.16

Problem (P2P) 14.1

$$\left.\begin{array}{l} G^{(1)}=0,\ r=r_1, \\ G^{(1)}=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi; \left\{\begin{array}{l} U(r_{1`},\varphi')=s_1(r_{1`},\varphi') \\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi') \end{array}\right.$$
$$\left.\begin{array}{l} G^{(1)}=0,\ \varphi=0, \\ G^{(1)}=0,\ \varphi=\pi \end{array}\right\}, r_1\le r\le r_2; \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi)=s_4(\rho,\pi). \end{array}\right.$$

Answer to Problem (P2P) 14.1 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.2

$$\left.\begin{array}{l} G^{(2)}=0,\ r=r_1, \\ \partial G^{(2)}/\partial n_2=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi; \left\{\begin{array}{l} U(r_{1`},\varphi')=s_1(r_{1`},\varphi'); \\ \partial U(r_{2`},\varphi')/\partial n_2=g_2(r_2,\varphi); \end{array}\right.$$
$$\left.\begin{array}{l} G^{(2)}=0,\ \varphi=0, \\ G^{(2)}=0,\ \varphi=\pi \end{array}\right\}, r_1\le r\le r_2; \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi)=s_4(\rho,\pi). \end{array}\right.$$

Answer to Problem (P2P) 14.2 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.3

$$\left.\begin{array}{l} \partial G^{(3)}/\partial n_1=0,\ r=r_1, \\ G^{(3)}=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi; \left\{\begin{array}{l} \partial U(r_{1`},\varphi')/\partial n_1=g_1(r_{1`},\varphi'); \\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi'); \end{array}\right.$$
$$\left.\begin{array}{l} G^{(3)}=0,\ \varphi=0, \\ G^{(3)}=0,\ \varphi=\pi \end{array}\right\}, r_1\le r\le r_2; \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi)=s_4(\rho,\pi). \end{array}\right.$$

Answer to Problem (P2P) 14.3 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.4

$$\left.\begin{array}{l}\partial G^{(4)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(4)}/\partial n_2 = 0,\ r = r_2,\end{array}\right\} 0 \le \varphi \le \pi; \left\{\begin{array}{l}\partial U(r_{1`}, \varphi^{/})/\partial n_1 = g_1(r_{1`}, \varphi^{/});\\ \partial U(r_{2`}, \varphi^{/})/\partial n_2 = g_2(r_2, \varphi);\end{array}\right.$$

$$\left.\begin{array}{l}G^{(4)} = 0,\ \varphi = 0,\\ G^{(4)} = 0,\ \varphi = \pi\end{array}\right\}, r_1 \le r \le r_2; \qquad \left\{\begin{array}{l}U(\rho,0) = s_3(\rho,0);\\ U(\rho,\pi) = s_4(\rho,\pi).\end{array}\right.$$

Answer to Problem (P2P) 14.4 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.5

$$\left.\begin{array}{l}\partial G^{(5)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(5)}/\partial n_2 = 0,\ r = r_2\end{array}\right\}, 0 \le \varphi \le \pi; \left\{\begin{array}{l}\partial U(r_{1`}, \varphi^{/})/\partial n_1 = g_1(r_{1`}, \varphi^{/});\\ \partial U(r_{2`}, \varphi^{/})/\partial n_2 = g_2(r_2, \varphi);\end{array}\right.$$

$$\left.\begin{array}{l}\partial G^{(5)}/\partial n_{\varphi_0} = 0,\ \varphi = 0,\\ \partial G^{(5)}/\partial n_{\varphi_\pi} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2; \left\{\begin{array}{l}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_4(\rho,\pi);\end{array}\right.$$

Answer to Problem (P2P) 14.5 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.6

$$\left.\begin{array}{l}G^{(6)} = 0,\ r = r_1,\\ \partial G^{(6)}/\partial n_2 = 0,\ r = r_2\end{array}\right\}, 0 \le \varphi \le \pi; \quad \left\{\begin{array}{l}U(r_{1`}, \varphi^{/}) = s_1(r_{1`}, \varphi^{/});\\ \partial U(r_{2`}, \varphi^{/})/\partial n_2 = g_2(r_2, \varphi);\end{array}\right.$$

$$\left.\begin{array}{l}\partial G^{(6)}/\partial n_{\varphi_0} = 0,\ \varphi = 0,\\ \partial G^{(6)}/\partial n_{\varphi_\pi} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2; \quad \left\{\begin{array}{l}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_4(\rho,\pi).\end{array}\right.$$

Answer to Problem (P2P) 14.6 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.7

$$\left.\begin{array}{l}\partial G^{(7)}/\partial n_1 = 0,\, r = r_1,\\ G^{(7)} = 0,\, r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi; \qquad \begin{cases}\partial U\left(r_{1`}, \varphi^{/}\right)/\partial n_1 = g_1\left(r_{1`}, \varphi^{/}\right);\\ U\left(r_{2`}, \varphi^{/}\right) = s_2\left(r_{2`}, \varphi^{/}\right);\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(7)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ \partial G^{(7)}/\partial n_{\varphi_\pi} = 0,\, \varphi = \pi\end{array}\right\} r_1 \le r \le r_2; \qquad \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.7 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.8

$$\left.\begin{array}{l}G^{(8)} = 0,\, r = r_1,\\ G^{(8)} = 0,\, r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi; \qquad \begin{cases}U\left(r_{1`}, \varphi^{/}\right) = s_1\left(r_{1`}, \varphi^{/}\right);\\ U\left(r_{2`}, \varphi^{/}\right) = s_2\left(r_{2`}, \varphi^{/}\right);\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(8)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ \partial G^{(8)}/\partial n_{\varphi_\pi} = 0,\, \varphi = \pi\end{array}\right\} r_1 \le r \le r_2; \qquad \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.8 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.9

$$\left.\begin{array}{l}\partial G^{(9)}/\partial n_1 = 0,\, r = r_1,\\ \partial G^{(9)}/\partial n_2 = 0,\, r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi; \qquad \begin{cases}\partial U\left(r_{1`}, \varphi^{/}\right)/\partial n_1 = g_1\left(r_{1`}, \varphi^{/}\right);\\ \partial U\left(r_{2`}, \varphi^{/}\right)/\partial n_2 = g_2\left(r_2, \varphi\right);\end{cases}$$

$$\left.\begin{array}{l}G^{(9)} = 0, \varphi = 0,\\ \partial G^{(9)}/\partial n_{\varphi_\pi} = 0,\, \varphi = \pi\end{array}\right\} r_1 \le r \le r_2; \qquad \begin{cases}U(\rho,0) = s_3(\rho,0);\\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi} = g_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.9 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.10

$$\left.\begin{array}{l} G^{(10)}=0,\ r=r_1, \\ \partial G^{(10)}/\partial n_2=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi; \qquad \left\{\begin{array}{l} U\left(r_{1`},\varphi^{/}\right)=s_1\left(r_{1`},\varphi^{/}\right); \\ \partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2=g_2\left(r_2,\varphi\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(10)}=0,\ \varphi=0, \\ \partial G^{(10)}/\partial n_{\varphi_\pi}=0,\ \varphi=\pi \end{array}\right\} r_1\le r\le r_2; \qquad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi}=g_4(\rho,\pi); \end{array}\right.$$

Answer to Problem (P2P) 14.10 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.11

$$\left.\begin{array}{l} \partial G^{(11)}/\partial n_1=0,\ r=r_1, \\ G^{(11)}=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi; \left\{\begin{array}{l} \partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right); \\ U\left(r_{2`},\varphi^{/}\right)=s_2\left(r_{2`},\varphi^{/}\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(11)}=0,\ \varphi=0, \\ \partial G^{(11)}/\partial n_{\varphi_\pi}=0,\ \varphi=\pi \end{array}\right\} r_1\le r\le r_2; \qquad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi}=g_4(\rho,\pi). \end{array}\right.$$

Answer to Problem (P2P) 14.11 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.12

$$\left.\begin{array}{l} G^{(12)}=0,\ r=r_1, \\ G^{(12)}=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi; \qquad \left\{\begin{array}{l} U\left(r_{1`},\varphi^{/}\right)=s_1\left(r_{1`},\varphi^{/}\right); \\ U\left(r_{2`},\varphi^{/}\right)=s_2\left(r_{2`},\varphi^{/}\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(12)}=0,\ \varphi=0, \\ \partial G^{(12)}/\partial n_{\varphi_\pi}=0,\ \varphi=\pi \end{array}\right\} r_1\le r\le r_2; \qquad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ \partial U(\rho,\pi)/\partial n_{\varphi_\pi}=g_4(\rho,\pi). \end{array}\right.$$

Answer to Problem (P2P) 14.12 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.13

$$\left.\begin{array}{l}\partial G^{(13)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(13)}/\partial n_2 = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi;\ \begin{cases}\partial U(r_{1`}, \varphi^{/})/\partial n_1 = g_1(r_{1`}, \varphi^{/});\\ \partial U(r_{2`}, \varphi^{/})/\partial n_2 = g_2(r_2, \varphi);\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(13)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(13)} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2;\ \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ U(\rho,\pi) = s_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.13 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.14

$$\left.\begin{array}{l}G^{(14)} = 0,\ r = r_1,\\ \partial G^{(14)}/\partial n_2 = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi;\ \begin{cases}U(r_{1`}, \varphi^{/}) = s_1(r_{1`}, \varphi^{/});\\ \partial U(r_{2`}, \varphi^{/})/\partial n_2 = g_2(r_2, \varphi);\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(14)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(14)} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2;\ \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ U(\rho,\pi) = s_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.14 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 14.15

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_1 = 0,\ r = r_1,\\ G^{(15)} = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi;\ \begin{cases}\partial U(r_{1`}, \varphi^{/})/\partial n_1 = g_1(r_{1`}, \varphi^{/});\\ U(r_{2`}, \varphi^{/}) = s_2(r_{2`}, \varphi^{/});\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(15)} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2;\ \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ U(\rho,\pi) = s_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.15 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

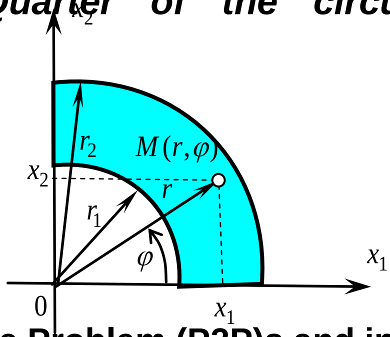
Problem (P2P) 14.16

$$\left.\begin{array}{l}G^{(16)} = 0,\ r = r_1,\\ G^{(16)} = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \pi;\ \begin{cases}U(r_{1`}, \varphi^{/}) = s_1(r_{1`}, \varphi^{/});\\ U(r_{2`}, \varphi^{/}) = s_2(r_{2`}, \varphi^{/});\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(16)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(16)} = 0,\ \varphi = \pi\end{array}\right\} r_1 \le r \le r_2;\ \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ U(\rho,\pi) = s_4(\rho,\pi).\end{cases}$$

Answer to Problem (P2P) 14.16 for half of the circular layer (Green's function for Poisson's equation in polar coordinates)

(P2P) 15. Quarter of the circular layer $(r_1 \le r \le r_2, \quad 0 \le \varphi \le \pi/2)$



15.1 Boundary-value Problem (P2P)s and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{15.1}$$

in the inner points of the circle. On the it boundary are given two of the following four functions:

$$\begin{gathered}
\partial U(r_{1`},\varphi')/\partial n_1 = g_1(r_{1`},\varphi') \ \ or \ \ U(r_{1`},\varphi') = s_1(r_{1`},\varphi') \ and \\
\partial U(r_{2`},\varphi')/\partial n_2 = g_2(r_2,\varphi) \quad or \quad U(r_{2`},\varphi') = s_2(r_{2`},\varphi'). \\
\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0); \quad U(\rho,0) = s_3(\rho,0); \\
\partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}} = g_4(\rho,\pi/2); \quad U(\rho,\pi/2) = s_4(\rho,\pi/2).
\end{gathered} \tag{15.2}$$

The solutions of these boundary value Problem (P2P)s are expressed via respective Green functions in the form of following integral formula:

$$U(r,\varphi)=\int_{r_1}^{r_2}\int_0^{\pi/2} f(\rho,\psi)G(r,\rho;\varphi,\psi)\rho d\rho d\varphi +$$
$$\int_0^{\pi/2}\left[\frac{\partial U(r_1,\varphi^{/})}{\partial n_1}G(r_1,r;\varphi^{/},\varphi)-U(r_1,\varphi^{/})\frac{\partial G(r_1,r;\varphi^{/},\varphi)}{\partial n_1}\right]r_1 d\varphi^{/} +$$
$$\int_0^{\pi/2}\left[\frac{\partial U(r_2,\varphi^{/})}{\partial n_2}G(r_2,r;\varphi^{/},\varphi)-U(r_2,\varphi^{/})\frac{\partial G(r_2,r;\varphi^{/},\varphi)}{\partial n_2}\right]r_2 d\varphi^{/} + \quad (15.3)$$
$$\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}}G(\rho,r;0,\varphi)-U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right]d\rho +$$
$$\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,\pi/2)}{\partial n_{\varphi_{\pi/2}}}G(\rho,r;\pi/2,\varphi)-U(\rho,\pi/2)\frac{\partial G(\rho,r;\pi/2,\varphi)}{\partial n_{\varphi_{\pi/2}}}\right]d\rho$$
.

In eq.(15.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi)=-\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi)=-f(r,\varphi) \quad (15.4)$$

and the following boundary conditions:

Problems (P2P) 15.1- 15.16

Problem (P2P) 15.1

$$\left.\begin{array}{l} G^{(1)}=0,\ r=r_1, \\ G^{(1)}=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi/2; \quad \left\{\begin{array}{l} U(r_{1`},\varphi^{/})=s_1(r_{1`},\varphi^{/}); \\ U(r_{2`},\varphi^{/})=s_2(r_{2`},\varphi^{/}); \end{array}\right.$$
$$\left.\begin{array}{l} G^{(1)}=0,\ \varphi=0, \\ G^{(1)}=0,\ \varphi=\pi/2 \end{array}\right\}, r_1\le r\le r_2; \quad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi/2)=s_4(\rho,\pi/2). \end{array}\right.$$

Answer to Problem (P2P) 15.1 for quarter of the circular layer (Green’s function for Poisson’s equation in polar coordinates)

Problem (P2P) 15.2

$$\left.\begin{array}{l} G^{(2)}=0,\ r=r_1, \\ \partial G^{(2)}/\partial n_2=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi/2; \quad \left\{\begin{array}{l} U\left(r_{1`},\varphi^{/}\right)=s_1\left(r_{1`},\varphi^{/}\right); \\ \partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2=g_2\left(r_2,\varphi\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(2)}=0,\varphi=0, \\ G^{(2)}=0,\ \varphi=\pi/2 \end{array}\right\}, r_1\le r\le r_2; \quad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi/2)=s_4(\rho,\pi/2). \end{array}\right.$$

Answer to Problem (P2P) 15.2 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.3

$$\left.\begin{array}{l} \partial G^{(3)}/\partial n_1=0,\ r=r_1, \\ G^{(3)}=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi/2; \quad \left\{\begin{array}{l} \partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right); \\ U\left(r_{2`},\varphi^{/}\right)=s_2\left(r_{2`},\varphi^{/}\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(3)}=0,\varphi=0, \\ G^{(3)}=0,\ \varphi=\pi/2 \end{array}\right\}, r_1\le r\le r_2; \quad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi/2)=s_4(\rho,\pi/2). \end{array}\right.$$

Answer to Problem (P2P) 15.3 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.4

$$\left.\begin{array}{l} \partial G^{(4)}/\partial n_1=0,\ r=r_1, \\ \partial G^{(4)}/\partial n_2=0,\ r=r_2, \end{array}\right\} 0\le\varphi\le\pi/2; \quad \left\{\begin{array}{l} \partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right); \\ \partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2=g_2\left(r_2,\varphi\right); \end{array}\right.$$

$$\left.\begin{array}{l} G^{(4)}=0,\varphi=0, \\ G^{(4)}=0,\ \varphi=\pi/2 \end{array}\right\}, r_1\le r\le r_2; \quad \left\{\begin{array}{l} U(\rho,0)=s_3(\rho,0); \\ U(\rho,\pi/2)=s_4(\rho,\pi/2). \end{array}\right.$$

Answer to Problem (P2P) 15.4 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.5

$$\left.\begin{array}{l} \partial G^{(5)}/\partial n_1=0,\ r=r_1, \\ \partial G^{(5)}/\partial n_2=0,\ r=r_2 \end{array}\right\}, 0\le\varphi\le\pi/2; \quad \left\{\begin{array}{l} \partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right); \\ \partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2=g_2\left(r_2,\varphi\right); \end{array}\right.$$

$$\left.\begin{array}{l} \partial G^{(5)}/\partial n_{\varphi_0}=0,\varphi=0, \\ \partial G^{(5)}/\partial n_{\varphi_{\pi/2}}=0,\ \varphi=\pi/2 \end{array}\right\} r_1\le r\le r_2; \quad \left\{\begin{array}{l} \partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0); \\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}}=g_4(\rho,\pi/2); \end{array}\right.$$

Answer to Problem (P2P) 15.5 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.6

$$\left.\begin{array}{l} G^{(6)}=0,\ r=r_1, \\ \partial G^{(6)}/\partial n_2=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi/2;\quad \left\{\begin{array}{l} U(r_{1`},\varphi')=s_1(r_{1`},\varphi'); \\ \partial U(r_{2`},\varphi')/\partial n_2=g_2(r_2,\varphi); \end{array}\right.$$

$$\left.\begin{array}{l} \partial G^{(6)}/\partial n_{\varphi_0}=0,\varphi=0, \\ \partial G^{(6)}/\partial n_{\varphi_{\pi/2}}=0,\ \varphi=\pi/2 \end{array}\right\} r_1\le r\le r_2;\quad \left\{\begin{array}{l} \partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0); \\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}}=g_4(\rho,\pi/2); \end{array}\right.$$

Answer to Problem (P2P) 15.6 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.7

$$\left.\begin{array}{l} \partial G^{(7)}/\partial n_1=0,\ r=r_1, \\ G^{(7)}=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi/2;\quad \left\{\begin{array}{l} \partial U(r_{1`},\varphi')/\partial n_1=g_1(r_{1`},\varphi'); \\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi'); \end{array}\right.$$

$$\left.\begin{array}{l} \partial G^{(7)}/\partial n_{\varphi_0}=0,\varphi=0, \\ \partial G^{(7)}/\partial n_{\varphi_{\pi/2}}=0,\ \varphi=\pi/2 \end{array}\right\} r_1\le r\le r_2;\quad \left\{\begin{array}{l} \partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0); \\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}}=g_4(\rho,\pi/2); \end{array}\right.$$

Answer to Problem (P2P) 15.7 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.8

$$\left.\begin{array}{l} G^{(8)}=0,\ r=r_1, \\ G^{(8)}=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\pi/2;\quad \left\{\begin{array}{l} U(r_{1`},\varphi')=s_1(r_{1`},\varphi'); \\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi'); \end{array}\right.$$

$$\left.\begin{array}{l} \partial G^{(8)}/\partial n_{\varphi_0}=0,\varphi=0, \\ \partial G^{(8)}/\partial n_{\varphi_{\pi/2}}=0,\ \varphi=\pi/2 \end{array}\right\} r_1\le r\le r_2;\left\{\begin{array}{l} \partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0); \\ \partial U(\rho,\pi/2)/\partial n_{\varphi_{\pi/2}}=g_4(\rho,\pi/2); \end{array}\right.$$

Answer to Problem (P2P) 15.8 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.9

$$\left.\begin{aligned} &\partial G^{(9)}/\partial n_1 = 0,\ r = r_1, \\ &\partial G^{(9)}/\partial n_2 = 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} \partial U(r_{1`}, \varphi')/\partial n_1 = g_1(r_{1`}, \varphi'); \\ \partial U(r_{2`}, \varphi')/\partial n_2 = g_2(r_2, \varphi); \end{cases}$$

$$\left.\begin{aligned} &G^{(9)} = 0, \varphi = 0, \\ &\partial G^{(9)}/\partial n_{\varphi_{\pi/2}} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \quad \begin{cases} U(\rho, 0) = s_3(\rho, 0); \\ \partial U(\rho, \pi/2)/\partial n_{\varphi_{\pi/2}} = g_4(\rho, \pi/2); \end{cases}$$

Answer to Problem (P2P) 15.9 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.10

$$\left.\begin{aligned} &G^{(10)} = 0,\ r = r_1, \\ &\partial G^{(10)}/\partial n_2 = 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} U(r_{1`}, \varphi') = s_1(r_{1`}, \varphi'); \\ \partial U(r_{2`}, \varphi')/\partial n_2 = g_2(r_2, \varphi); \end{cases}$$

$$\left.\begin{aligned} &G^{(10)} = 0, \varphi = 0, \\ &\partial G^{(10)}/\partial n_{\varphi_{\pi/2}} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \quad \begin{cases} U(\rho, 0) = s_3(\rho, 0); \\ \partial U(\rho, \pi/2)/\partial n_{\varphi_{\pi/2}} = g_4(\rho, \pi/2); \end{cases}$$

Answer to Problem (P2P) 15.10 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.11

$$\left.\begin{aligned} &\partial G^{(11)}/\partial n_1 = 0,\ r = r_1, \\ &G^{(11)} = 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} \partial U(r_{1`}, \varphi')/\partial n_1 = g_1(r_{1`}, \varphi'); \\ U(r_{2`}, \varphi') = s_2(r_{2`}, \varphi'); \end{cases}$$

$$\left.\begin{aligned} &G^{(11)} = 0, \varphi = 0, \\ &\partial G^{(11)}/\partial n_{\varphi_{\pi/2}} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \quad \begin{cases} U(\rho, 0) = s_3(\rho, 0); \\ \partial U(\rho, \pi/2)/\partial n_{\varphi_{\pi/2}} = g_4(\rho, \pi/2); \end{cases}$$

Answer to Problem (P2P) 15.11 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.12

$$\left.\begin{aligned} G^{(12)} &= 0,\ r = r_1, \\ G^{(12)} &= 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} U(r_{1`}, \varphi') = s_1(r_{1`}, \varphi'); \\ U(r_{2`}, \varphi') = s_2(r_{2`}, \varphi'); \end{cases}$$

$$\left.\begin{aligned} & G^{(12)} = 0, \varphi = 0, \\ & \partial G^{(12)} / \partial n_{\varphi_{\pi/2}} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \begin{cases} U(\rho, 0) = s_3(\rho, 0); \\ \partial U(\rho, \pi/2) / \partial n_{\varphi_{\pi/2}} = g_4(\rho, \pi/2); \end{cases}$$

Answer to Problem (P2P) 15.12 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.13

$$\left.\begin{aligned} \partial G^{(13)} / \partial n_1 &= 0,\ r = r_1, \\ \partial G^{(13)} / \partial n_2 &= 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} \partial U(r_{1`}, \varphi') / \partial n_1 = g_1(r_{1`}, \varphi'); \\ \partial U(r_{2`}, \varphi') / \partial n_2 = g_2(r_2, \varphi); \end{cases}$$

$$\left.\begin{aligned} & \partial G^{(13)} / \partial n_{\varphi_0} = 0, \varphi = 0, \\ & G^{(13)} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \quad \begin{cases} \partial U(\rho, 0) / \partial n_{\varphi_0} = g_3(\rho, 0); \\ U(\rho, \pi/2) = s_4(\rho, \pi/2). \end{cases}$$

Answer to Problem (P2P) 15.13 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.14

$$\left.\begin{aligned} & G^{(14)} = 0,\ r = r_1, \\ & \partial G^{(14)} / \partial n_2 = 0,\ r = r_2 \end{aligned}\right\},\ 0 \le \varphi \le \pi/2; \quad \begin{cases} U(r_{1`}, \varphi') = s_1(r_{1`}, \varphi'); \\ \partial U(r_{2`}, \varphi') / \partial n_2 = g_2(r_2, \varphi); \end{cases}$$

$$\left.\begin{aligned} & \partial G^{(14)} / \partial n_{\varphi_0} = 0, \varphi = 0, \\ & G^{(14)} = 0,\ \varphi = \pi/2 \end{aligned}\right\} r_1 \le r \le r_2; \quad \begin{cases} \partial U(\rho, 0) / \partial n_{\varphi_0} = g_3(\rho, 0); \\ U(\rho, \pi/2) = s_4(\rho, \pi/2). \end{cases}$$

Answer to Problem (P2P) 15.14 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.15

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_1=0,\ r=r_1,\\ G^{(15)}=0,\ r=r_2\end{array}\right\},\ 0\le\varphi\le\pi/2;\qquad \begin{cases}\partial U(r_{1`},\varphi')/\partial n_1=g_1(r_{1`},\varphi');\\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi');\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_{\varphi_0}=0,\varphi=0,\\ G^{(15)}=0,\ \varphi=\pi/2\end{array}\right\}\ r_1\le r\le r_2;\qquad \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0);\\ U(\rho,\pi/2)=s_4(\rho,\pi/2).\end{cases}$$

Answer to Problem (P2P) 15.15 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 15.16

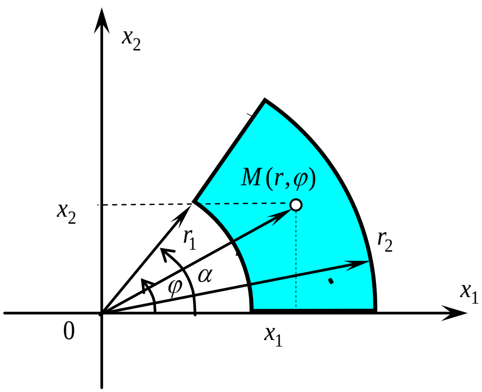
$$\left.\begin{array}{l}G^{(16)}=0,\ r=r_1,\\ G^{(16)}=0,\ r=r_2\end{array}\right\},\ 0\le\varphi\le\pi/2;\qquad \begin{cases}U(r_{1`},\varphi')=s_1(r_{1`},\varphi');\\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi');\end{cases}$$

$$\left.\begin{array}{l}\partial G^{(16)}/n_{\varphi_0}=0,\varphi=0,\\ G^{(16)}=0,\ \varphi=\pi/2\end{array}\right\}\ r_1\le r\le r_2;\ \begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0);\\ U(\rho,\pi/2)=s_4(\rho,\pi/2).\end{cases}$$

Answer to Problem (P2P) 15.16 for quarter of the circular layer (Green's function for Poisson's equation in polar coordinates)

(P2P) ***16. Sector of the circular layer***

$(r_1\le r\le r_2,\quad 0\le\varphi\le\alpha)$



16.1 Boundary-value Problems (P2P) and integral representation via Green's functions

Let us consider the following boundary value Problem (P2P)s which consist from the Poisson equation

$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \tag{16.1}$$

in the inner points of the circle. On the it boundary are given four of the following eight functions:

$$\begin{gathered}
\partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1 = g_1\left(r_{1`},\varphi^{/}\right) \;\; or \;\; U\left(r_{1`},\varphi^{/}\right) = s_1\left(r_{1`},\varphi^{/}\right) and \\
\partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2 = g_2\left(r_2,\varphi\right) \quad or \quad U\left(r_{2`},\varphi^{/}\right) = s_2\left(r_{2`},\varphi^{/}\right). \\
\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0); \quad U(\rho,0) = s_3(\rho,0); \\
\partial U(\rho,\alpha)/\partial n_{\varphi_\alpha} = g_4(\rho,\alpha); \quad U(\rho,\alpha) = s_4(\rho,\alpha).
\end{gathered} \tag{16.2}$$

The solutions of these boundary value Problems (P2P) are expressed via respective Green functions in the form of following integral formula:

$$\begin{gathered}
U(r,\varphi) = \int_{r_1}^{r_2}\int_0^{\alpha} f(\rho,\psi) G(r,\rho;\varphi,\psi)\rho d\rho d\varphi + \\
\int_0^{\alpha}\left[\frac{\partial U\left(r_1,\varphi^{/}\right)}{\partial n_1} G\left(r_1,r;\varphi^{/},\varphi\right) - U\left(r_1,\varphi^{/}\right)\frac{\partial G\left(r_1,r;\varphi^{/},\varphi\right)}{\partial n_1}\right] r_1 d\varphi^{/} + \\
\int_0^{\alpha}\left[\frac{\partial U\left(r_2,\varphi^{/}\right)}{\partial n_2} G\left(r_2,r;\varphi^{/},\varphi\right) - U\left(r_2,\varphi^{/}\right)\frac{\partial G\left(r_2,r;\varphi^{/},\varphi\right)}{\partial n_2}\right] r_2 d\varphi^{/} + \\
\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,0)}{\partial n_{\varphi_0}} G(\rho,r;0,\varphi) - U(\rho,0)\frac{\partial G(\rho,r;0,\varphi)}{\partial n_{\varphi_0}}\right] d\rho + \\
\int_{r_1}^{r_2}\left[\frac{\partial U(\rho,\alpha)}{\partial n_{\varphi_\alpha}} G(\rho,r;\alpha,\varphi) - U(\rho,\pi/2)\frac{\partial G(\rho,r;\alpha,\varphi)}{\partial n_{\varphi_\alpha}}\right] d\rho.
\end{gathered} \tag{16.3}$$

In eq.(16.3) of this section the Green functions G and functions U satisfy Poisson Equations

$$\nabla^2 G(r,\rho;\varphi,\psi) = -\delta(r-\rho)\delta(\varphi-\psi);$$
$$\nabla^2 U(r,\varphi) = -f(r,\varphi) \qquad (16.4)$$

and the following boundary conditions:

Problems (P2P) 16.1- (P2P) 16.16

Problem (P2P) 16.1

$$\left.\begin{aligned} G^{(1)} &= 0, r = r_1, \\ G^{(1)} &= 0, r = r_2, \end{aligned}\right\} 0 \le \varphi \le \alpha; \quad \begin{cases} U(r_{1`},\varphi') = s_1(r_{1`},\varphi'); \\ U(r_{2`},\varphi') = s_2(r_{2`},\varphi'); \end{cases}$$
$$\left.\begin{aligned} G^{(1)} &= 0, \varphi = 0, \\ G^{(1)} &= 0, \varphi = \alpha \end{aligned}\right\}, r_1 \le r \le r_2; \quad \begin{cases} U(\rho,0) = s_3(\rho,0); \\ U(\rho,\alpha) = s_4(\rho,\alpha). \end{cases}$$

Answer to Problem (P2P) 16.1 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.2

$$\left.\begin{aligned} G^{(2)} &= 0, r = r_1, \\ \partial G^{(2)}/\partial n_2 &= 0, r = r_2, \end{aligned}\right\} 0 \le \varphi \le \alpha; \quad \begin{cases} U(r_{1`},\varphi') = s_1(r_{1`},\varphi'); \\ \partial U(r_{2`},\varphi')/\partial n_2 = g_2(r_2,\varphi); \end{cases}$$
$$\left.\begin{aligned} G^{(2)} &= 0, \varphi = 0, \\ G^{(2)} &= 0, \varphi = \alpha \end{aligned}\right\}, r_1 \le r \le r_2; \quad \begin{cases} U(\rho,0) = s_3(\rho,0); \\ U(\rho,\alpha) = s_4(\rho,\alpha). \end{cases}$$

Answer to Problem (P2P) 16.2 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.3

$$\left.\begin{aligned} \partial G^{(3)}/\partial n_1 &= 0, r = r_1, \\ G^{(3)} &= 0, r = r_2, \end{aligned}\right\} 0 \le \varphi \le \alpha; \quad \begin{cases} \partial U(r_{1`},\varphi')/\partial n_1 = g_1(r_{1`},\varphi'); \\ U(r_{2`},\varphi') = s_2(r_{2`},\varphi'); \end{cases}$$
$$\left.\begin{aligned} G^{(3)} &= 0, \varphi = 0, \\ G^{(3)} &= 0, \varphi = \alpha \end{aligned}\right\}, r_1 \le r \le r_2; \quad \begin{cases} U(\rho,0) = s_3(\rho,0); \\ U(\rho,\alpha) = s_4(\rho,\alpha). \end{cases}$$

Answer to Problem (P2P) 16.3 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.4

$$\left.\begin{aligned}\partial G^{(4)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(4)}/\partial n_2 = 0,\ r = r_2,\end{aligned}\right\}0 \le \varphi \le \alpha; \quad \left\{\begin{aligned}\partial U(r_{1`},\varphi')/\partial n_1 = g_1(r_{1`},\varphi');\\ \partial U(r_{2`},\varphi')/\partial n_2 = g_2(r_2,\varphi);\end{aligned}\right.$$

$$\left.\begin{aligned}G^{(4)} = 0,\ \varphi = 0,\\ G^{(4)} = 0,\ \varphi = \alpha\end{aligned}\right\}, r_1 \le r \le r_2; \quad \left\{\begin{aligned}U(\rho,0) = s_3(\rho,0);\\ U(\rho,\alpha) = s_4(\rho,\alpha).\end{aligned}\right.$$

Answer to Problem (P2P) 16.4 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.5

$$\left.\begin{aligned}\partial G^{(5)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(5)}/\partial n_2 = 0,\ r = r_2\end{aligned}\right\}, 0 \le \varphi \le \alpha; \quad \left\{\begin{aligned}\partial U(r_{1`},\varphi')/\partial n_1 = g_1(r_{1`},\varphi');\\ \partial U(r_{2`},\varphi')/\partial n_2 = g_2(r_2,\varphi);\end{aligned}\right.$$

$$\left.\begin{aligned}\partial G^{(5)}/\partial n_{\varphi_0} = 0,\ \varphi = 0,\\ \partial G^{(5)}/\partial n_{\varphi_\alpha} = 0,\ \varphi = \alpha\end{aligned}\right\} r_1 \le r \le r_2; \quad \left\{\begin{aligned}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha} = g_4(\rho,\alpha).\end{aligned}\right.$$

Answer to Problem (P2P) 16.5 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.6

$$\left.\begin{aligned}G^{(6)} = 0,\ r = r_1,\\ \partial G^{(6)}/\partial n_2 = 0,\ r = r_2\end{aligned}\right\}, 0 \le \varphi \le \alpha; \quad \left\{\begin{aligned}U(r_{1`},\varphi') = s_1(r_{1`},\varphi');\\ \partial U(r_{2`},\varphi')/\partial n_2 = g_2(r_2,\varphi);\end{aligned}\right.$$

$$\left.\begin{aligned}\partial G^{(6)}/\partial n_{\varphi_0} = 0,\ \varphi = 0,\\ \partial G^{(6)}/\partial n_{\varphi_\alpha} = 0,\ \varphi = \alpha\end{aligned}\right\} r_1 \le r \le r_2; \quad \left\{\begin{aligned}\partial U(\rho,0)/\partial n_{\varphi_0} = g_3(\rho,0);\\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha} = g_4(\rho,\alpha).\end{aligned}\right.$$

Answer to Problem (P2P) 16.6 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.7

$$\left.\begin{aligned}&\partial G^{(7)}/\partial n_1=0,\ r=r_1,\\&G^{(7)}=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad\begin{cases}\partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right);\\U\left(r_{2`},\varphi^{/}\right)=s_2\left(r_{2`},\varphi^{/}\right);\end{cases}$$

$$\left.\begin{aligned}&\partial G^{(7)}/\partial n_{\varphi_0}=0,\varphi=0,\\&\partial G^{(7)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}r_1\le r\le r_2;\quad\begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0);\\\partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.7 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.8

$$\left.\begin{aligned}&G^{(8)}=0,\ r=r_1,\\&G^{(8)}=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad\begin{cases}U\left(r_{1`},\varphi^{/}\right)=s_1\left(r_{1`},\varphi^{/}\right);\\U\left(r_{2`},\varphi^{/}\right)=s_2\left(r_{2`},\varphi^{/}\right);\end{cases}$$

$$\left.\begin{aligned}&\partial G^{(8)}/\partial n_{\varphi_0}=0,\varphi=0,\\&\partial G^{(8)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}r_1\le r\le r_2;\quad\begin{cases}\partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0);\\\partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.8 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.9

$$\left.\begin{aligned}&\partial G^{(9)}/\partial n_1=0,\ r=r_1,\\&\partial G^{(9)}/\partial n_2=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad\begin{cases}\partial U\left(r_{1`},\varphi^{/}\right)/\partial n_1=g_1\left(r_{1`},\varphi^{/}\right);\\\partial U\left(r_{2`},\varphi^{/}\right)/\partial n_2=g_2(r_2,\varphi);\end{cases}$$

$$\left.\begin{aligned}&G^{(9)}=0,\varphi=0,\\&\partial G^{(9)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}r_1\le r\le r_2;\quad\begin{cases}U(\rho,0)=s_3(\rho,0);\\\partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.9 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.10

$$\left.\begin{aligned} &G^{(10)}=0,\ r=r_1,\\ &\partial G^{(10)}/\partial n_2=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad \begin{cases}U(r_{1`},\varphi')=s_1(r_{1`},\varphi');\\ \partial U(r_{2`},\varphi')/\partial n_2=g_2(r_2,\varphi);\end{cases}$$

$$\left.\begin{aligned} &G^{(10)}=0,\ \varphi=0,\\ &\partial G^{(10)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}\ r_1\le r\le r_2;\quad \begin{cases}U(\rho,0)=s_3(\rho,0);\\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.10 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.11

$$\left.\begin{aligned} &\partial G^{(11)}/\partial n_1=0,\ r=r_1,\\ &G^{(11)}=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad \begin{cases}\partial U(r_{1`},\varphi')/\partial n_1=g_1(r_{1`},\varphi');\\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi');\end{cases}$$

$$\left.\begin{aligned} &G^{(11)}=0,\ \varphi=0,\\ &\partial G^{(11)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}\ r_1\le r\le r_2;\quad \begin{cases}U(\rho,0)=s_3(\rho,0);\\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.11 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.12

$$\left.\begin{aligned} &G^{(12)}=0,\ r=r_1,\\ &G^{(12)}=0,\ r=r_2\end{aligned}\right\},\ 0\le\varphi\le\alpha;\quad \begin{cases}U(r_{1`},\varphi')=s_1(r_{1`},\varphi');\\ U(r_{2`},\varphi')=s_2(r_{2`},\varphi');\end{cases}$$

$$\left.\begin{aligned} &G^{(12)}=0,\ \varphi=0,\\ &\partial G^{(12)}/\partial n_{\varphi_\alpha}=0,\ \varphi=\alpha\end{aligned}\right\}\ r_1\le r\le r_2;\quad \begin{cases}U(\rho,0)=s_3(\rho,0);\\ \partial U(\rho,\alpha)/\partial n_{\varphi_\alpha}=g_4(\rho,\alpha).\end{cases}$$

Answer to Problem (P2P) 16.12 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.13

$$\left.\begin{array}{l}\partial G^{(13)}/\partial n_1 = 0,\ r = r_1,\\ \partial G^{(13)}/\partial n_2 = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \alpha; \quad \left\{\begin{array}{l}\partial U\left(r_{1`}, \varphi'\right)/\partial n_1 = g_1\left(r_{1`}, \varphi'\right);\\ \partial U\left(r_{2`}, \varphi'\right)/\partial n_2 = g_2\left(r_2, \varphi\right);\end{array}\right.$$

$$\left.\begin{array}{l}\partial G^{(13)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(13)} = 0,\ \varphi = \alpha\end{array}\right\} r_1 \le r \le r_2; \quad \left\{\begin{array}{l}\partial U(\rho, 0)/\partial n_{\varphi_0} = g_3(\rho, 0);\\ U(\rho, \alpha) = s_4(\rho, \alpha).\end{array}\right.$$

Answer to Problem (P2P) 16.13 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.14

$$\left.\begin{array}{l}G^{(14)} = 0,\ r = r_1,\\ \partial G^{(14)}/\partial n_2 = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \alpha; \quad \left\{\begin{array}{l}U\left(r_{1`}, \varphi'\right) = s_1\left(r_{1`}, \varphi'\right);\\ \partial U\left(r_{2`}, \varphi'\right)/\partial n_2 = g_2\left(r_2, \varphi\right);\end{array}\right.$$

$$\left.\begin{array}{l}\partial G^{(14)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(14)} = 0,\ \varphi = \alpha\end{array}\right\} r_1 \le r \le r_2; \quad \left\{\begin{array}{l}\partial U(\rho, 0)/\partial n_{\varphi_0} = g_3(\rho, 0);\\ U(\rho, \alpha) = s_4(\rho, \alpha).\end{array}\right.$$

Answer to Problem (P2P) 16.14 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.15

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_1 = 0,\ r = r_1,\\ G^{(15)} = 0,\ r = r_2\end{array}\right\},\ 0 \le \varphi \le \alpha; \quad \left\{\begin{array}{l}\partial U\left(r_{1`}, \varphi'\right)/\partial n_1 = g_1\left(r_{1`}, \varphi'\right);\\ U\left(r_{2`}, \varphi'\right) = s_2\left(r_{2`}, \varphi'\right);\end{array}\right.$$

$$\left.\begin{array}{l}\partial G^{(15)}/\partial n_{\varphi_0} = 0, \varphi = 0,\\ G^{(15)} = 0,\ \varphi = \alpha\end{array}\right\} r_1 \le r \le r_2; \quad \left\{\begin{array}{l}\partial U(\rho, 0)/\partial n_{\varphi_0} = g_3(\rho, 0);\\ U(\rho, \alpha) = s_4(\rho, \alpha).\end{array}\right.$$

Answer to Problem (P2P) 16.15 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)

Problem (P2P) 16.16

$$\left.\begin{array}{l} G^{(16)}=0,\ r=r_1, \\ G^{(16)}=0,\ r=r_2 \end{array}\right\},\ 0\le\varphi\le\alpha;\quad \begin{cases} U\left(r_1,\varphi'\right)=s_1\left(r_1,\varphi'\right); \\ U\left(r_2,\varphi'\right)=s_2\left(r_2,\varphi'\right); \end{cases}$$

$$\left.\begin{array}{l} \partial G^{(16)}/\partial n_{\varphi_0}=0,\ \varphi=0, \\ G^{(16)}=0,\ \varphi=\alpha \end{array}\right\}\ r_1\le r\le r_2;\ \begin{cases} \partial U(\rho,0)/\partial n_{\varphi_0}=g_3(\rho,0); \\ U(\rho,\alpha)=s_4(\rho,\alpha). \end{cases}$$

Answer to Problem (P2P) 16.16 for sector of the circular layer (Green's function for Poisson's equation in polar coordinates)