The ΘG-Convolution Method for Green's Integral Formulas Derivation

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ABSTRACT

Proposed in this paper method is based on the general integral formula for thermoelastic influence functions

$$U_k(x,x) = g \int_V \Theta^{(k)}(z,x) G(x,z) dV(z); x,z,x \in V \tag{1}$$

and on the generalized Green's integral formula in thermoelasticity

$$\begin{aligned} u_k(x) = a^{-1} \int_V F(x) U_k(x,x) dV(x) - \int_{\Gamma_D} T(y) \frac{\partial U_k(y,x)}{\partial n_y} d\Gamma_D(y) + \\ + \int_{\Gamma_N} \frac{\partial T(y)}{\partial n_y} U_k(y,x) d\Gamma_N(y) + \int_{\Gamma_M} \left[T(y) + \frac{a}{a} \frac{\partial T(y)}{\partial n_y} \right] U_k(y,x) d\Gamma_M(y), \end{aligned} \tag{2}$$

published earlier. The main difficulties for its practical realization are the derivation of the functions of influence of a unit concentrated forces onto elastic volume dilatation $\Theta^{(k)}$, of the Green's functions in heat conduction G and, also, of the computation of some convolutions over domain of the product of these functions. Due mentioned above specific difficulties it was called the ΘG -convolution Method (ΘG-CM). For canonical Cartesian domains the mentioned above functions $\Theta^{(k)}$ and G have been derived successfully for hundreds boundary-value problems (BVP) and were presented in an earlier published handbook. So, it can be confirmed that the ΘG-CM will open a great really possibility to derive thermoelastic influence functions and Green's type integral formulas for many new BVP in thermoelasticity for Cartesian canonical domains. As example of such kind results, in this paper are presented, in closed form, the functions of influence of unit point heat source onto displacements and a Green's type integral formula for a BVP for thermoelastic half-plane, formulated in the form of **a theorem about deriving the Green's integral formula.**

Let the field of displacements $u_k(x); k = 1,2$ *in the inner points* $x \equiv (x_1, x_2)$ *of the elastic half-plane* $V(0 \le x_1 < \infty; -\infty < x_2 < \infty)$ *is determined by no homogeneous Lame equations*

$$m\nabla^2 u_k(x) + (l + m) q_{,k}(x) - g T_{,k}(x) = 0 \tag{3}$$

and in the points $y \equiv (0, y_2)$ *of its boundary straight line* $\Gamma(y_1 = 0; -\infty < y_2 < \infty)$ *the following*

homogeneous mechanical conditions (of sliding fixation type) are given:

$$u_1(x_1 = 0, x_2 = y_{2,}) = s_{12}(x_1 = 0, x_2 = y_{2,}) = 0, \quad (4)$$

where s_{12} *are the tangential stresses, determined by the Hooke's law.*
Let also the temperature field $T(x)$ *in Eq. (3), generated by the inner heat source* $F(x)$ *and temperature* $T(y) \in \Gamma$ *(Dirichlet's boundary condition), satisfies to the following BVP in heat conduction*

$$\nabla^2 T(x) = -a^{-1} F(x), x \in V; \quad T(x_1 = 0, x_2 = y_{2,}) = T(y_2). \quad (5)$$

Then the displacements $u_k(x)$ *of this BVP in Eqs (3)-(5) of thermoelasticity for considered half-plane can be presented by the following Green's integral formula, written in the matrix form:*

$$\mathbf{u}(\xi) = \frac{1}{a}\int_0^\infty \int_{-\infty}^\infty F(x)\mathbf{U}(\mathbf{x},\xi) dx_1 dx_2 - \int_{-\infty}^\infty T(y_2)\mathbf{Q}(\mathbf{y}_2,\xi) dy_2 + d_{2k} c; \; k = 1,2, \quad (6)$$

where the influence matrices $\mathbf{U}(\mathbf{x},\xi)$ *and* $\mathbf{Q}(\mathbf{y}_2,\xi)$ *of an inner unit point heat source and of a unit point temperature on the* Γ *onto thermoelastic displacements, also the desired displacements matrix* $\mathbf{u}(\xi)$, *are determined as follows:*

$$\mathbf{U}(\mathbf{x},\xi) = \begin{pmatrix} U_1(x,\xi) \\ U_2(x,\xi) \end{pmatrix} = \frac{g}{8p(l+2m)} \begin{pmatrix} 2(x_1 - x_1)\ln(r_1/r) \\ 2(x_2 - x_2)\ln(r_1/r) - 4x_1 arctg[(x_1 + x_1)/(x_2 - x_2)] \end{pmatrix}; \begin{pmatrix} r = \sqrt{(x_1 - x_1)^2 + (x_2 - x_2)^2}; \\ r_1 = \sqrt{(x_1 + x_1)^2 + (x_2 - x_2)^2} \end{pmatrix};$$

$$\mathbf{Q}(\mathbf{y}_2,\xi) = \begin{pmatrix} Q_1(y_3, x) \\ Q_2(y_2, x) \end{pmatrix} = -\frac{g}{2p(l+2m)} \begin{pmatrix} x_1 \dfrac{x_1}{r^2} \\ -(y_2 - x_2)\dfrac{x_1}{r^2} + arctg[(x_1)/(y_2 - x_2)] \end{pmatrix}; \; \mathbf{u}(\xi) = \begin{pmatrix} u_1(\xi) \\ u_2(\xi) \end{pmatrix}; r = \sqrt{x_1^2 + (y_2 - x_2)^2}, \quad (7)$$

where $g = a_t(2m + 3l)$; a_t *– the coefficient of the linear thermal dilatation;* l, m *– Lame's constants of elasticity; a – is the coefficient of temperature conductivity;* d_{2k} *is the Kronecker's symbol and* c *is an arbitrary constant.*

Conclusions: 1/. Obtained in a closed form expressions for thermoelastic influence functions and the Green's integral formula in Eqs (6) and (7) are very useful for practical numerical implementation. These results can be used both in boundary integral equation method (BIEM) (thermoelastic influence functions as a kernels) and in boundary element method (BEM) to create an efficient boundary element. 2/.The proposed here ΘG-CM will also really work for any orthogonal canonical domains, as soon as the lists of the influence functions $\Theta^{(k)}$ and Green's functions G will be completed. The ΘG-CM is applicable for solution of the BVP not only in thermoelasticity, but also in some particular cases of electroelasticity, magnetoelasticity and poroelasticity.

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